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A study on the feasibility of combinatorial measures

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Abstract: *The cornerstone and heart of financial market risk management is risk measurement. The size of market risk and the possibility of a risk occurrence must be determined before issuing risk analysis reports, hedging or diversifying to transfer risks, or setting and changing risk capital limitations. This is important when making decisions about risk management, portfolio optimization, and asset allocation. This study suggests a combination of risk measurement:* $\alpha\rho_1(X) + (1-\alpha)\rho_2(X)$, *which takes into account all the traits of various measurement approaches, depicts the traits of risk from various angles, and provides a thorough index for financial institutions to use when calculating risk. Meanwhile, convex combinations of multi-class risk measures also provide a flexible way to consider investors' risk appetite.*

Keywords: *Convex combination; Expected shortfall; Information entropy-standard deviation model; Combined risk measures*

I. Introduction

One of the most important ideas in risk management is the convex portfolio. Convex combination of various risk metrics might result in a more thorough and precise risk assessment [1]. Combined risk measurement: $\alpha\rho_1(X) + (1-\alpha)\rho_2(X)$ comprehensively considers the characteristics of different types of measurement, depicts the characteristics of risk in multiple aspects, and gives a comprehensive index for financial institutions when measuring risk. And set the parameters according to the actual financial market fluctuations [2]. By adjusting the weights of different risk measures, risk management strategies can be tailored to the risk tolerance and preferences of specific investors. This helps investors balance risk and return more effectively and create a personalized investment plan based on their goals and constraints. The study of combinatorial measures can promote the development of risk measurement theory. By exploring the convex combination relationship between different risk measures, we can deeply understand the nature and characteristics of risk. This is important for improving existing risk measurement methods, proposing new risk measurement frameworks, and deepening the understanding of core concepts of risk management.

In conclusion, the research on convex combination of risk measurement is of great significance for risk management, investment decision-making and the development of risk theory. It helps investors assess and comprehend risks more comprehensively, optimize portfolio allocation, and develop risk management strategies that adapt to individual needs. In addition, the research on convex combination also provides further exploration and development direction for risk measurement theory.

II. Verification of the feasibility of combined measures

2.1Numerical example of expected shortfall and information entropy-standard deviation combination measure

Under normal circumstances, the yield of financial assets is a leptokurtic heavy tail distribution, which cannot follow with the normal distribution that is usually used. Some unexpected events with greater harm but lower probability may cause huge losses. Understanding the probability distribution of extreme events is conducive to avoiding risks. The extreme value theory model considers the distribution of the tail values of the sequence and fits the data in the sequence that exceeds a given threshold, which is consistent with the concept of managing the risk of low-frequency high-hazard events.

2.2 VAR and ES based on extreme value models

The generalized Pareto distribution was proposed by Pikands [3] in 1975, and its distribution function has two important parameters ξ and σ, ξ is the shape parameter, σ is the scale parameter [4] ,$Y = X - u$, u is the given threshold, and the model is as follows:

$$F_u(y) = P(X - u \leq y | X > u), 0 \leq y \leq x_F - u,$$

where $y = X - u$ represents the portion that exceeds the threshold and x_F is the right endpoint of the distribution function F. $F_u(y)$ can [5] also be written as

$$F_u(y) = \frac{F(u+y) - F(u)}{1 - F(u)} = \frac{F(X) - F(u)}{1 - F(u)}.$$

Using the Pikands-Balkama-deHaan theorem, the excess number distribution can be fitted by a generalized Pareto distribution. Thus the distribution of the tail data of the sequence can be written as

$$F(x) = \big(1 - F(u)\big) G_{\xi,\sigma}(y) + F(u),$$

$F(u)$can be replaced by a frequency of the number of samples less than the threshold, combined with the generalized Pareto distribution function to obtain

$$F(x) = 1 - \frac{N_u}{n}\left[1 + \xi\left(\frac{x-u}{\sigma}\right)\right]^{-\xi}.$$

In the measure of risk, the concept of VaR [6] is that given the confidence level, the maximum loss that a certain asset portfolio may occur in the future period of time, using extreme value theory to estimate VaR is a very effective method, when [7] the given probability is p, $VaR_p = F^{-1}(p)$, substitute $F(x)$ to get

$$VaR_p = u + \frac{\sigma}{\xi}\left[\left(\frac{n}{N_u}(1 - p)\right)^{-\xi} - 1\right].$$

The part of the loss level that exceeds VaR is measured by the expected shortfall model, which is as follows

$$ES_p = E\big(X \big| X \geq VaR_p\big).$$

Expected shortfall formula under the POT model are given by [8]

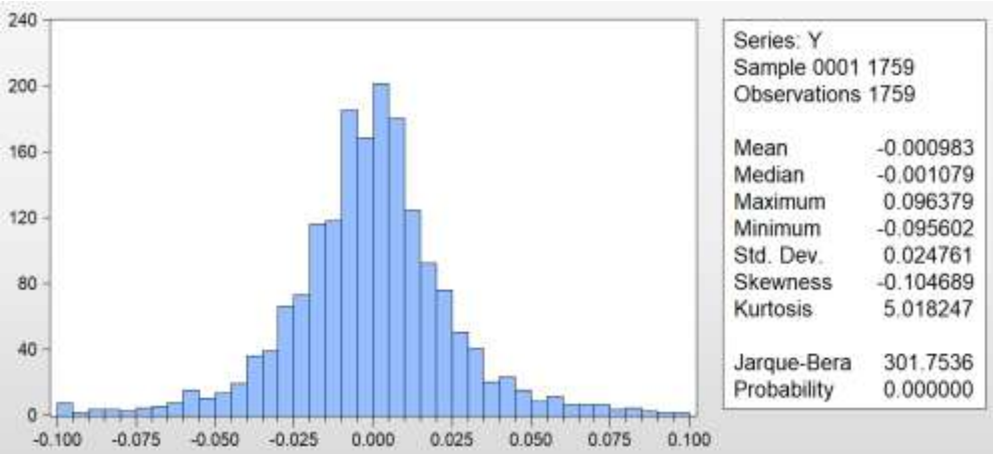
$$ES_p = \frac{VaR_p}{1-\xi} + \frac{\sigma - \xi u}{1-\xi}.$$

2.3 Data processing

This paper selects the daily closing price p_t of Shaanxi coal stocks, a total of 1759 data, and takes the data logarithmic to obtain a new series, according to the yield formula:

$$r_t = ln(p_t) - ln(p_{t-1}).$$

The yield series of the stock is calculated. When studying VaR and CVaR in this paper, the yield series is taken as the opposite number for ease of understanding. First, analyze the distribution of the data, the histogram is shown in the figure below:

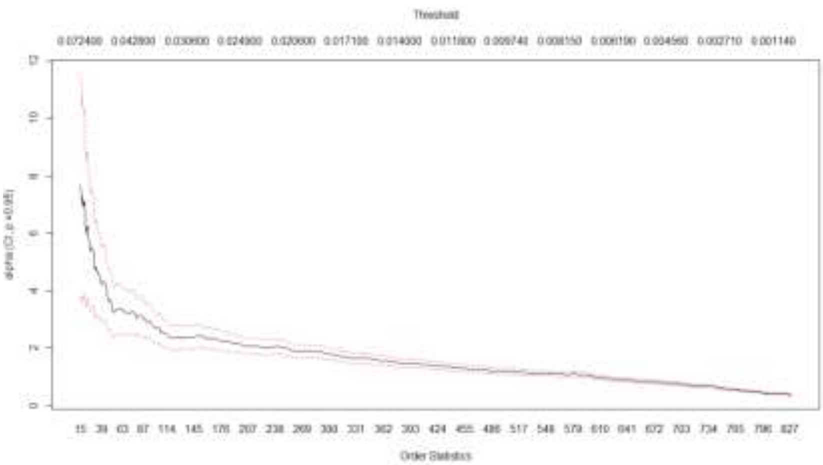


Through the histogram, it can be seen that the sequence does show the characteristics of spikes and thick tails, with a kurtosis value of 5.018247, more than 3, and a skewness of -0.104689, which has negative skew characteristics, which also indicates that the data do not conform to the normal distribution.

2.4 Thresholds

The yield series presents the characteristics of peak thick tail, focusing on the tail risk of the yield series, which is the basis for studying the risk measurement of extreme value theory. To confirm the rationality of applying extreme value theory instead of normal distribution to study the yield of stock index futures, it is necessary to test whether the yield series is normally distributed. Jarque_Bera test [9] value is 301.7535, which means that the tail data of the yield does not conform to the normal distribution, and it is reasonable to fit it with the extreme value distribution model.

The threshold for determining the sequence can be taken using a Hill plot, as shown in the following figure:



According to the theory of Reiss and Thomas, the tail data of the sequence will decrease and eventually stabilize as the amount of data increases [10]. By finding the end of the line fluctuating in the Hill plot, you can determine the number of data that exceeds the threshold. From the figure, it can be determined that after the 108th data, the Hill curve gradually stabilizes. The corresponding threshold is 0.034606, and there are 107 data exceeding the threshold.

2.5 GPD parameter estimation and tail risk measurement

In the case of threshold determination, the shape parameters and scale parameters in the POT model can be estimated using the sample data. Using the maximum likelihood estimation method, the parameter estimation is performed using the MATLAB program, and the results are as follows:

Parameter	Estimates
Shape parameters $\hat{\xi}$	-0.2476
Scale parameters $\hat{\sigma}$	0.0240
Fixed threshold$u$	0.0346

Through the previous POT model has obtained the formula of VaR and expected shortfall, substitute the shape parameter ξ and scale parameter σ obtained by the maximum likelihood estimation method, and the determined threshold u, you can determine the specific VaR and expected shortfall size at a certain confidence level, the list is as follows:

Confidence level	VaR	ES
0.99	0.069548	0.081850
0.975	0.053761	0.069196
0.95	0.039199	0.057524

From the data in the table, it can be seen that the value of the expected shortfall is greater than the value at risk, which is also consistent with the theoretical basis.

2.6 Information entropy-standard deviation model

Shannon, the father of information theory in the 40s of the 20th century, introduced it into the category of information theory and named information entropy to measure the average amount of information from sources. From this information, entropy [11] was also introduced into the ranks of measuring risk, and the academic community began a boom in research. Information entropy is based on the amount of information. The amount of information is a measure of how much information it has. Because the probability of a random event occurring, the likelihood of an event in each state being included is also different.

The model of the amount of information in the discrete state [12] is:

$\phi(p_i) = lnp_i,$

where X is a discrete random variable, $P(X = x_i) = p_i$, $0 \leq p_i \leq 1(i = 1,2,\ldots n)$, $\sum_{i=1}^{n} p_i = 1$ .The information entropy of discrete event X is obtained:

$H = -k\sum_{i=1}^{n} p_i lnp_i.$

By comparing the information entropy measure and variance, Li Yinghua believes that the variance can only express the degree of deviation of random variables from expectations, and cannot determine the overall change of random variables, while the information entropy measure is more comprehensive in the measurement of risk from probability, and the information entropy measure is regarded as variance information supplement proposed the information entropy-standard deviation model, the specific model is $R(X) = \lambda H_\theta(X) + (1-\lambda)S_\theta(X)$, where $R(X)$ represents the risk of portfolio$X$, $H_\theta(X)$ represents the information entropy of the portfolio in state, and $S_\theta(X)$ represents the standard deviation of the portfolio. $0 < \lambda \leq 1,\lambda$ is the risk appetite coefficient of the decision-maker [13].

In order to obtain the value of the information entropy measure, it is necessary to know the distribution law of the yield series. Since the yield distribution ranges from -10% to 10%, it is divided into 40 intervals with a range length of 0.5%. According to Bernoulli's law of large numbers, when the amount of data is sufficient, the frequency converges to probability according to probability. After calculation, the value of the information entropy-standard deviation model under different risk appetite conditions is obtained, as shown in the following table:

Risk appetite coefficient	Information entropy measure	Standard Deviation	Information entropy-standard deviation measure
0.5	2.952036	0.024761	1.4883985
0.7	2.952036	0.024761	2.0738535

Using the rate of return as a random variable, the variance can be calculated after understanding the distribution, but the general variance can only reflect the fluctuation of the yield above and below the mean, and cannot reflect the overall return.

Combined with information entropy measures, you can get a comprehensive picture of risk events. At the same time, by controlling the risk appetite coefficient, the specific meaning of the measure can be automatically adjusted. In the case given, the larger the risk appetite coefficient, the more the measurement results reflect the overall uncertainty.

III. Combinatorial measure analysis

On the basis of the known two candidate measures, the expected shortfall and the information entropy-standard deviation measure, the combined risk measure model is given by
$\rho(X) = \alpha ES + (1-\alpha)R(X)$.

Using the POT model and generalized Pareto distribution in extreme value distribution theory, the risk of stock returns in Shaanxi coal industry in low-frequency and high-risk events was studied, which measures the average loss under an extreme situation. The standard deviation, as the square root of the variance, emphasizes the effect of positive and negative deviation on risk. The information entropy-standard deviation measure can both reflect the overall uncertainty of the random variable and show the degree of deviation from the mean. When selecting 0.5parameters, decision-makers balance the size of extreme losses with the volatility of the overall risk. The information entropy-standard deviation measure under the expected shortfall and neutral risk appetite at the confidence level of 0.99 is selected, and the values of the combined measure are shown in the following table:

Combined parameterα	ES	Information entropy-standard deviation measure	Combined risk measures
0.3	0.082107	1.149715	0.8294326
0.5	0.082107	1.149715	0.6159110
0.7	0.082107	1.149715	0.4023894

The combination of the two candidate measures can reflect both loss and return changes as well as volatility. By combining expected shortfall and information entropy-standard deviation, both mean and variance can be considered, providing a more comprehensive risk assessment that balances the relationship between risk and reward. It not only looks at the possible loss of a portfolio or asset, but also considers the potential return that comes from this risk. This helps investors consider risk and reward to make more informed investment decisions. Select different parametersα to express different preferences. For the parameter level, when $0 < \alpha \leq 0.5$, financial institutions are risk-averse in decision-making, and pay more attention to the overall change and volatility of random variables. When $0.5 < \alpha \leq 1$, decision-makers prefer risk, and portfolio risk measures focus more on losses to be suffered in extreme events than just volatility.

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