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Study on the Value of Liabilities Influenced by Multi-period Capital Requirements

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ABSTRACT: *The aim of this study is to explore the value of corporate liabilities influenced by multi-period capital requirements and to construct a new liability valuation framework. The research steps are as follows: We first combine other relevant research and option pricing theory, and fully consider the real situation to construct a new liability valuation framework, under which we obtain the iterative pricing formula for corporate liabilities. Then we assume that the future price of risky assets follows a log-normal distribution, and on this basis design a digital simulation to verify the effectiveness of the liability valuation framework*

***Keywords*：** *Corporate Liability Value, Multi-period Capital Requirements, Risk-neutral Pricing, Geometric Brownian Motion.*

I. INTRODUCTION

The liability situation of Chinese listed companies has four main characteristics: First, the overall level of liabilities is gradually rising, indicating a trend of expanding corporate debt scale. Second, the proportion of current liabilities is high and the proportion of non-current liabilities is low, indicating that companies face greater short-term debt pressure and relatively less long-term debt. Third, the proportion of payables and advance receipts is gradually increasing, and the proportion of short-term loans is gradually decreasing, reflecting the increase in transactions between companies and suppliers and customers and the pursuit of more stable financing sources. Fourth, the owner's equity ratio is still relatively high, indicating that the company still has relatively high net assets. According to data released by the National Bureau of Statistics, as of the end of 2021, the total assets of industrial enterprises above designated size were 141.29 trillion yuan, an increase of 9.9% over the previous year; total liabilities were 79.23 trillion yuan, an increase of 9.6%; total owner's equity was 62.06 trillion yuan, an increase of 10.2%; The asset-liability ratio was 56.1%, a decrease of 0.1 percentage point from the previous year. China's corporate debt market is currently facing a historic transformation. Administrative control is gradually being relaxed, and the market will become the main force constraining the development of the corporate debt market. With the market-oriented development of the corporate bond market, various risks, especially credit risks, will be significantly magnified. Therefore, we need to strengthen research on corporate debt risk pricing mechanisms to ensure that risks and returns are reasonably shared between investors and enterprises, thereby enhancing market efficiency.

Asset pricing theory is one of the important areas of research in finance and one of the most systematic and fruitful areas in finance research. The 1960s and 1970s were the golden age of asset pricing theory development, during which time we saw the emergence of asset pricing models that are still well-known to us today. For example, Sharpe proposed the Capital Asset Pricing Model (CAPM), which helps investors determine the expected return and risk level of a portfolio by calculating the expected return and risk level of a market portfolio. Ross proposed the Arbitrage Pricing Theory (APT), which explains changes in asset returns through multi-factor analysis. Black and Scholes proposed option pricing theory, which provides an important theoretical basis for option trading. After the 1980s, the rise of behavioral finance broke the category of traditional finance and incorporated human psychological behavior and real constraints into the framework of asset pricing. These emerging theories provide us with a more comprehensive and deeper understanding of financial markets and investment decisions.

With the enrichment of asset pricing theory, the research on corporate liability value has gradually deepened. From the cash flow discount model and relative value evaluation model of Fisher in the 20th century, it has gradually developed to structured models, mainly including The Longstaff and Schwartz Model, The Merton Model, The Geske Model, The Leland and Toft Model, The Briys-de Varenne Model, etc. All these models assume that: the fluctuation of corporate asset value is a random process; when the net assets of the company are lower than the debt owed, the company's bonds default; bonds can only be executed or not executed. These models have made great contributions to promoting corporate debt pricing, but their shortcomings are also obvious. One is that the assumptions are too simple and deviate greatly from reality. The second is that the calculation is too complicated.

Through the study of structured models and the research of Engsner and others in the field of insurance liabilities, we can find that most of the previous structured models focus on the "passive" default after the company reaches the default conditions. For example, the Merton model assumes that when the company's asset value is lower than its liabilities, the company goes bankrupt, and does not emphasize the subjective ability of shareholders in this process. In combination with reality, we find that a specific scenario may be that in the case of continuous expansion of debt scale, high stock liabilities, and high liquidity tension, huge dividends are still implemented. This is called Ponzi dividends. In China, the concept of Ponzi dividends originated from Xie Deren (2013) published "Theoretical Research on Corporate Dividend Ability". In this article, he first conducted in-depth research on the definition and measurement of free cash flow, and on this basis proposed the concept of corporate dividend ability, that is, whether the company has the ability to continuously distribute corporate free cash to shareholders within the positive boundary of retained earnings. He believes that corporate dividend ability is constrained by free cash derived from free cash flow and double boundaries of retained earnings. Therefore, when a company has no dividend ability and pays dividends, its essence is Ponzi dividends.

Based on the above basic definition of Ponzi dividends, we can find that in the process of whether a company defaults, in addition to the influencing factors of corporate operations, the attitude of corporate shareholders towards dividends is also crucial. A typical example is that according to public data, China Evergrande Group has paid dividends every year from 2016 to 2020, accumulating to 939.33 billion yuan. Among them, in 2020, the year before Evergrande fell into a debt crisis, its dividends reached as high as 577.79 billion yuan, accounting for 61.5% of the total dividends. Therefore, this paper fully combines the reality to construct a two-stage liability value evaluation model affected by multi-period capital requirements. This model incorporates the key factor of shareholder dividends into it and reflects the subjective initiative of shareholders in the process of corporate bankruptcy in the first stage of the model, realizing the reasonable and accurate pricing of corporate liabilities under discrete conditions. These research results provide financial decision-makers with an effective means to deal with uncertainty problems.

II. VALUATION FRAMEWORK

This chapter mainly proposes a more realistic valuation framework for corporate liabilities based on the structured model and the valuation method of Engsner and others in the field of insurance liabilities, using the principle of risk-neutral pricing. The main idea is: to divide the company's assets into risky assets and risk-free assets, risk-free assets are to meet the capital requirements of each period. If a company wants to continue to operate, it must meet the interest generated by paying off liabilities and the capital requirements of the next period. Excess funds can be distributed to shareholders through dividends, and shareholders also have the right to inject capital into the company to meet operational requirements. Shareholders have the right to choose when to stop operating, determine the cash flow obtained by creditors through the time when shareholders stop operating, and discount it.

2.1 Basic Assumptions

In the Merton model, when a company's asset value is lower than its total liabilities, the company will go bankrupt. However, in reality, many insolvent listed companies have not gone bankrupt, such as Jiangxi Zheng bang Technology Co., Ltd. and Shanghai Quan zhu Holdings Group Co., Ltd. There are mainly two reasons for this: one is due to the mismatch of asset and liability terms, as long as the company's current assets are greater than current liabilities, the company can solve the debt pressure in the short term; the second is debt restructuring, which can be understood as the company's shareholders, creditors or third parties investing to save the company.

Multi-period capital requirements refer to the minimum level of capital that a company needs to retain in each period during its operation to cope with various risks (such as credit risk, market risk, operational risk, etc.). Regulatory authorities in various countries will also set corresponding capital adequacy ratio standards according to different industries to ensure the stable and sustainable development of enterprises. The main influencing factors include: type of business, operating capital, employee compensation, etc.

This paper proposes the following specific assumptions:

1、The company' s assets are divided into risky assets and risk-free assets. Risk-free assets are to meet the capital requirements of each period. Risky assets are not only limited to financial assets, but can also be machinery and equipment, company projects, etc. All liabilities of the company are simplified into a debt that needs to repay principal and interest.

2、It is assumed that the owners of the company bear limited liability, so they can choose not to provide funds for the reference enterprise at any time in the future - the right to default.

3、It is assumed that the market is arbitrage-free, does not consider the impact of taxes, and multi-period capital requirements are given externally.

4、Once the risky assets are determined, it is not allowed to change the type and proportion of its internal assets, but shareholders can adjust the position of risky assets by buying and selling their quantity.

2.2 Liability Valuation Framework

We consider the time periods $1,\dots,T$, which correspond to the time points $0,1,\dots,T$, and a filtered probability space $(\Omega,\mathcal{F},\mathbb{F},\mathbb{Q})$, where $\mathbb{F}=(\mathcal{F}_t)_{t=0}^{T}$ is a filter that satisfies $\{\emptyset,\Omega\}=\mathcal{F}_0\subseteq\cdots\subseteq\mathcal{F}_T=\mathcal{F}$, and $\mathbb{Q}$ represents the risk-neutral measure. We use $L^p(\mathcal{F}_t,\mathbb{Q})$ to represent the normed linear space of $\mathcal{F}_t$-measurable random variables X with norm $\mathbb{E}^{\mathbb{Q}}[|X|^p]^{1/p}$.In addition, when two random variables are equal or one random variable is greater than another random variable, this paper refers to the case where these equations or inequalities hold with a probability of 1. That is to say, these equations or inequalities are almost certain to hold, and there may be very few cases where they do not hold. In this paper, we define $x^+:=max(0,x)$, which means taking the larger of x and 0, and $x^-:=min(0,x)$, which means taking the smaller of x and 0. The sum of the empty set is $\sum_{k=1}^{0}a_k=0$, and the product of the empty set is:$\prod_{k=1}^{0}a_k=1$.

Assume there is a company whose assets are divided into risky assets and risk-free assets. Let S_t represent the risky assets and R_t represent the risk-free assets, where t represents time. In addition, the company has a liability that needs to repay principal and interest and matures at time T. Let B represent the book value of the liability. Shareholders have the right to stop operations at any time, and continuous operations must pay interest and meet the capital requirements of the next period.

The cash flow generated by the risky asset in each period is

$$X_t^r=(e^d-1)S_t,\quad t\in\mathbb{A}$$

Where d represents the continuous compounding dividend rate of risky assets, and it is assumed that the d of each period is the same, $\mathbb{A}=\{1,\cdots,T\}$. It should be noted that d can be controlled by shareholders, because if the cash flow generated by assets is not as expected, shareholders can choose to sell part of the risky assets at fair value in the market to form cash flow. Conversely, shareholders can choose to buy risky assets. It should be noted that we did not consider the impact of taxes. If we consider the impact of taxes, because the tax system for stock capital gains and dividends in our country is not the same, shareholders cannot achieve conversion between equal capital gains and equal dividend profits by buying and selling assets. In addition, it should be noted that because one year is set as one period by default, the continuous compounding dividend rate d needs to be converted into a dividend rate compounded once a year.

The interest required by creditors each period is denoted as X^o, then the profit that shareholders can obtain from the company each period is:

$$c_t=\begin{cases}X_t^r-X^o+R_{t-1}-R_t, & t\in\mathbb{A}\\ X_t^r-X^o+R_{t-1}-R_t+S_t-B, & t=T\end{cases}.$$

Where c_t represents the profit obtained by shareholders each period.

At time $t = 1$, shareholders can choose to fulfill their obligations to creditors, or they can choose not to fulfill them. If the shareholder decides to default, it means giving up all rights and transferring the risk-free assets R_0 and risky assets S_1 to the creditors, where r represents the continuous compounding risk-free interest rate. Shareholders will neither receive any dividends nor suffer any losses due to the decision to default. If the shareholder chooses not to default, when $R_0 - R_1 - X_1^o + X_1^r \geq 0$, the positive surplus $R_0 - R_1 - X_1^o + X_1^r$ is paid to the shareholders, and the creditors receive X_1^o. When $R_0 - R_1 - X_1^o + X_1^r < 0$, shareholders must inject capital of $-R_0 + R_1 + X_1^o - X_1^r > 0$ to offset the deficit so that the company can continue to operate. In this case, X_1^o is also paid to creditors.

At time $t \in \{2, \dots, T\}$, shareholders choose to default or not to default, the situation is completely similar to the above situation at time $t = 1$.

Shareholders must determine a rule to determine under what circumstances a default will occur. The time point of shareholder default is called the stopping time τ, where $\tau \in \mathcal{S}_{1,T+1}$, $\mathcal{S}_{1,T+1}$ represents the set of stopping times, with a range of $\{1, \dots, T+1\}$. If $\tau = T + 1$, it means that shareholders will not default within the time period T.

The cumulative cash flow of shareholders can be written as:

$$C_0 = \begin{cases} \sum_{t=1}^{\tau-1} \left(\frac{R_{t-1} - R_t + X_t}{e^{rt}} \right), & \tau \in \mathbb{A} \\ \sum_{t=1}^{\tau-1} \left(\frac{R_{t-1} - R_t + X_t}{e^{rt}} \right) + \frac{S_{T+1} - B}{e^{rT}}, & \tau = T+1 \end{cases}$$

Where $X_t := X_t^r - X^o$ and $X_t^r = (e^d - 1)S_t$

According to the risk-neutral pricing theory, the value of this cash flow is:

$$\mathbb{E}^{\mathbb{Q}}[C_0] = \begin{cases} \sum_{t=1}^{\tau-1} \left(\frac{R_{t-1} - R_t + \mathbb{E}^{\mathbb{Q}}(X_t)}{e^{rt}} \right), & \tau \in \mathbb{A} \\ \sum_{t=1}^{\tau-1} \left(\frac{R_{t-1} - R_t + \mathbb{E}^{\mathbb{Q}}(X_t)}{e^{rt}} \right) + \frac{\mathbb{E}^{\mathbb{Q}}(S_{T+1}) - B}{e^{rT}},, & \tau = T+1 \end{cases}$$

Where $\mathbb{Q}$ represents the risk-neutral measure.

We assume that shareholders choose a stopping time τ that maximizes the value. In this case, the maximum value of C_0 at time 0, C_{max}, can be expressed as:

$$\begin{aligned} C_{max} &= \sup_{\tau \in \mathcal{S}_{1,T+1}} \mathbb{E}^{\mathbb{Q}}[C_0] \\ &= \sup_{\tau \in \mathcal{S}_{1,T+1}} \begin{cases} R_0 - \frac{R_{\tau-1}}{e^{r(\tau-1)}} - X^o \frac{1 - e^{-r(\tau-1)}}{e^r - 1} + S_0[1 - e^{-d(\tau-1)}], & \tau \in \mathbb{A} \\ R_0 - \frac{R_{\tau-1}}{e^{r(\tau-1)}} - X^o \frac{1 - e^{-r(\tau-1)}}{e^r - 1} + S_0\left[1 - e^{-d(\tau-1)}\right] + S_0 e^{-dT} - \frac{B}{e^{rT}},, & \tau = T+1 \end{cases} \end{aligned}$$

Next, let's analyze the value of the liability. First, we consider the situation where the creditor can obtain a cash flow in period T: if $S_T + R_{T-1}e^r > X^o + B$, the shareholder will not default, and the creditor obtains $X^o + B$; otherwise, the creditor obtains $S_T + R_{T-1}e^r$. Therefore,

$$I_T = min(S_T + R_{T-1}e^r, X^o + B)$$

The value of I_T in period $T - 1$ is:

$$\frac{\mathbb{E}^{\mathbb{Q}}(I_T)}{e^r}$$

Then, we consider the cash flow situation that creditors can obtain in period $T - 1$. If $S_{T-1} + R_{T-2} * e^r > X^o + \frac{\mathbb{E}^{\mathbb{Q}}(I_T)}{e^r}$, the shareholders will not default, and the creditors will get $X^o + \frac{\mathbb{E}^{\mathbb{Q}}(I_T)}{e^r}$. Otherwise, the creditors will get $S_{T-1} + R_{T-2} * e^r$. Therefore, we have:

$$I_{T-1} = min(S_{T-1} + R_{T-2} * \mathrm{e}^r, X^o + \frac{\mathbb{E}^{\mathbb{Q}}(I_T)}{\mathrm{e}^r})$$

Therefore, the recursive formula for I_τ can be obtained:

$$I_\tau = min\left(S_\tau + R_{\tau-1}\mathrm{e}^r, X^o + \frac{\mathbb{E}^{\mathbb{Q}}(I_{\tau+1})}{\mathrm{e}^r}\right), \tau \in \mathbb{A}$$

Where $S_{T+1} = B\mathrm{e}^r$.

Since the creditors receive interest X^o in each period before the shareholders default, the expected value of the creditors at the stopping time τ is:

$$V_\tau = X^o \sum_{t=1}^{\tau-1} \mathrm{e}^{rt} + \mathbb{E}^{\mathbb{Q}}(I_\tau) = X^o \frac{\mathrm{e}^r - \mathrm{e}^{r\tau}}{1 - \mathrm{e}^r} + \mathbb{E}^{\mathbb{Q}}(I_\tau), \tau \in \mathbb{A}$$

Next, we consider the cash flow that creditors can obtain in period $T-1$. If $S_{T-1} + R_{T-2} * \mathrm{e}^r > X^o + \frac{\mathbb{E}^{\mathbb{Q}}(I_T)}{\mathrm{e}^r}$, the shareholders will not default, and the creditors will get $X^o + \frac{\mathbb{E}^{\mathbb{Q}}(I_T)}{\mathrm{e}^r}$. Otherwise, the creditors will get $S_{T-1} + R_{T-2} * \mathrm{e}^r$. Therefore, we have:

$$I_{T-1} = min\left(S_{T-1} + R_{T-2} * \mathrm{e}^r, X^o + \frac{\mathbb{E}^{\mathbb{Q}}(I_T)}{\mathrm{e}^r}\right)$$

The recursive formula for I_τ can be obtained:

$$I_\tau = min(S_\tau + R_{\tau-1}\mathrm{e}^r, X^o + \frac{\mathbb{E}^{\mathbb{Q}}(I_{\tau+1})}{\mathrm{e}^r}), \tau \in \mathbb{A}$$

Where $S_{T+1} = B\mathrm{e}^r$.

Since the creditors receive interest X^o in each period before the shareholders default, the expected value of the creditors at the stopping time τ is:

$$V_\tau = X^o \sum_{t=1}^{\tau-1} \mathrm{e}^{rt} + \mathbb{E}^{\mathbb{Q}}(I_\tau) = X^o \frac{\mathrm{e}^r - \mathrm{e}^{r\tau}}{1 - \mathrm{e}^r} + \mathbb{E}^{\mathbb{Q}}(I_\tau), \tau \in \mathbb{A}$$

At the stopping time τ, the present value of the creditors is:

$$V_{\tau p} = \frac{V_\tau}{\mathrm{e}^\tau} = X^o \frac{1 - \mathrm{e}^{-r(\tau-1)}}{\mathrm{e}^r - 1} + \frac{\mathbb{E}^{\mathbb{Q}}(I_\tau)}{\mathrm{e}^\tau}, \tau \in \mathbb{A}$$

Where $V_{\tau p}$ represents the fair present value of the debt at time τ.

III. GEOMETRIC BROWNIAN MOTION AND NUMERICAL SIMULATION

Geometric Brownian Motion is a random process commonly used to simulate price changes in financial markets. We choose Geometric Brownian Motion as the standard random process, on the one hand, to keep consistent with the pricing of other financial products, such as European options. On the other hand, Geometric Brownian Motion has some excellent properties, such as: the price of risky assets will not be negative. At the same time, because the future price of risky assets of Geometric Brownian Motion follows a log-normal distribution, with the excellent properties of log-normal distribution, it can help us further simplify the formula for debt value. Therefore, in the following text, we will further analyze based on the assumption that the price of risky assets satisfies Geometric Brownian Motion.

First, we consider the first situation, that is, the necessary parameters are known to calculate the fair value of the company's liabilities. Suppose there is a company with an initial capital structure of 12 billion yuan in risky assets, 1 billion yuan in risk-free assets, and a book value of 10 billion yuan in debt. In addition, shareholders take 11.7% of the risky assets each period as their generated cash flow, and the company pays 909 million yuan in cash flow to creditors each period as interest, and the duration of this debt is 10 years. The risk-free interest rate is 5%. In addition, we obtain the capital requirements for the company to continue operations each period through a reasonable method. The initial capital requirement is 1 billion yuan, then it decreases by 100 million yuan each period until it ends at 0. That is:

Table 1: Company Data

Parameter	S_0	R_0	B	B	T	r	r	σ	$R_0 — R_T$
Value	120	10	100	9.09	10years	5%	11.7%	0.2	{10,… 1, 0}

We assume that the trend of risky asset prices follows the following geometric Brownian motion:

$$dS = (r - d)Sdt + \sigma SdW = (0.05 - 0.117)Sdt + 0.2SdW$$

Where W is the standard Brownian motion under the risk-neutral measure.

Table 2: Data Results

Parameter	C_{max}	$V^0_{\tau^*p}$	A	τ^*
Value	23.25	100.12	123.37	10

It should be noted that $V^0_{\tau^*p}$ has the same meaning as V_{τ^*p} mentioned earlier. According to the accounting identity:$Assets = Liabilities + Owner'sEquity$, we know that the initial owner's equity of the company is 30 billion yuan. According to formula , we can calculate the value of the default option owned by shareholders as 675 million yuan.

In addition, it is worth mentioning that when using Python programming, we adopted the method of Monte Carlo simulation. We generated a total of 100,000 log-normal distribution sample points to find the expected value of random variables. Since the samples themselves are random, the results obtained each time will have a small deviation, but these deviations are acceptable

IV. IN CONCLUSION

this paper has constructed a two-stage liability valuation model influenced by multi-period capital requirements, fully combining the actual situation. This model incorporates the key factor of shareholder dividends and reflects the subjective initiative of shareholders in the company's bankruptcy process in the first stage of the model, realizing the reasonable quasi-pricing of corporate liabilities in discrete situations. These research results provide financial decision-makers with an effective means to deal with uncertainty issues.

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REFERENCES

[1] Sharpe, W. F. (1964). *Capital asset prices: A theory of market equilibrium under conditions of risk*. The Journal of Finance, 19(3), 425-442.

[2] Ross, S. A. (1973). *Return, risk and arbitrage*. Rodney L. White Center for Financial Research, The Wharton School, University of Pennsylvania.

[3] Ross, S. A. (2013). *The arbitrage theory of capital asset pricing*. In Handbook of the fundamentals of financial decision making: Part I (pp. 11-30).

[4] Black, F., & Scholes, M. (1973). *The pricing of options and corporate liabilities*. Journal of Political Economy, 81(3), 637-654.

[5] Fisher, I. (1930). *The theory of interest*. New York, 43, 1-19.

[6] Graham, B., Dodd, D. L. F., Cottle, S., et al. (1962). *Security analysis: Principles and technique*. New York: McGraw-Hill.

[7] Fama, E. F., & French, K. R. (1992). *The cross- section of expected stock returns*. The Journal of Finance, 47(2), 427-465.

[8] Damodaran, A. (2012). *Investment valuation: Tools and techniques for determining the value of any asset*. Hoboken: John Wiley & Sons.

[9] Longstaff, F., & Schwartz, E. (1995). *Valuing risk debt: a new approach. Journal of Finance*, 789-820.

[10] Hull, J. (1997). *Option, Future, and Other Derivatives (3rd ed.)*. Prence Hall International Inc., pp. 100-351.

[11] Merton, R. (1974). *On the pricing of the corporate debt: the risk structure of interest rates*. Journal of Finance, 29, 449-470.

[12] Geske, R. (1997). *The valuation of corporate liabilities as compound option*. Journal of Financial and Quantitative Analysis, 541-552.

[13] Leland, H., & Toft, K. (1996). *Optimal capital structure, endogenous bankruptcy, and the term structure of credit spread*. Journal of Finance, 51, 987-1019.

[14] Eric Bris, Monzir Belarach, Hu Mingmai. (2002). *Options, Futures and Exotic Derivatives* [M]. Beijing: Machinery Industry Press.