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Optimal Advertising with Persistent Demand

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Abstract: *How much of a firm's budget should it devote to advertising? If it devotes too little money, the marginal cost of an additional advertising dollar will be below the gain from additional customers, causing it to miss out on a potentially substantial increase in revenue. If it devotes too much money, the firm ceases to reach new customers, and wastes money as a result. We study a model with stochastic demand shocks, where a firm aims to equalize marginal expenditures on advertising with expected gains from demand. We show that a firm that tracks previous patterns in demand significantly outperforms a firm that fails to do so. We also develop a model of persistent shocks and menu costs that can be generalized to further fields for future research.*

Keywords: *Advertising, Decision Theory, Economics, Marketing, Optimization JEL Classification Marketing and Advertising.*

I. Introduction

Advertising is an enormous industry, comprising 19% of US GDP last year [1]. In the modern era, rather than simply posting a newspaper advertisement, advertising campaigns are multifaceted. Techniques for advertising range from Google Ads to directed mailers. Firms utilize a myriad of communication sources in order to reach their target audience in the most effective manner. However, just as important as the question of how to advertise, is the question of how much to spend on advertising. The US Small Business Association recommends that companies allocate

"7 to 8 percent of your [their] gross revenue for marketing and advertising if you're [they're] doing less than $5 million a year in sales and your [their] net profit margin – after all expenses – is in the 10 percent to 12 percent range" [2]. However, what this advice fails to take into account is the firm's expected future revenue, using current revenue instead as a myopic prediction for the future. Indeed, a firm seeing rapid growth in sales would under spend should it use the above guideline with accounting for demand growth. Tailoring the level of advertising to demand is quite important. In this paper, we examine the real-world problem of a firm that aims to calibrate its level of advertising optimally. Overshooting involves excess expenditures that may fail to reach new convertible audiences, while undershooting leaves money on the table in the form of missed sales. Achieving the optimal level of advertising is essential to firms, as it allows them to achieve maximum efficiency when choosing how much to invest in advertising. We aim to devise a strategy that can be applied consistently over time to maximize firm profit from advertising. In order to do so, we develop a model of a firm facing stochastic demand. The firm seeks to optimize how often and to what extent it should change its level of advertising. However, changing its advertising scheme involves incurring a logistical cost each time the firm chooses to do so. Demand is modeled through a Markovian process and follows a persistent pattern. That is, if demand went up in the previous period, it is more likely to go up again the following period. Similarly, if demand went down in the previous period, it is more likely to go down again the following period. The firm responds to the level of demand by changing the level of advertising to match its expectation of future demand trends. We characterize the difference between the firm's profit maximizing strategy and a na¨ıve myopic strategy. We show that it is optimal for a firm to overshoot or undershoot when setting advertising levels at the end of a given period, conditional on the previous shift in demand. This follows due to the increased alignment between advertising and expected demand, given the persistence present. Thus, the optimal mechanism tracks patterns in demand and use those predictions to selectively set the level of advertising.

II. Literature Review

The study of environments with costly adjustment began with [3]. In their paper, Diamond examines the behavior of consumers when firms are aware of consumer demand. We differ here by considering the problem where the firm faces incomplete information and must commit ahead of time to changing the level of advertising. [4] study a scenario where inflation continually increases, causing posted firm prices to decrease automatically over time. For contrast, we study a problem where demand is imperfectly persistent, symmetrically positive or negative.

The importance of sticky pricing was shown in [5], which uses sticky prices to generate a model of the business cycle.

A great deal of empirical work has also studied the problem of price adjustment. [6] develop a model that numerically fits several empirical regularities. For instance, they find that price reductions are larger but less frequent than price increases. In a related paper, [7] test their asymmetrical predictions on manufacturing data, finding support for a menu cost model. [8] examine the practical costs behind price adjustment, importantly the practical costs don't simply equal physical menu costs, but rather include costs managerial costs. This motivates our decision to model advertising adjustment as a price adjustment problem in this paper.

III. Discrete Model

We study the decision problem of a firm that aims to invest in the correct level of advertising, in order to maximize profits. Time is discrete, $t \in \{1, 2, . . . , T\}$. The firm aims to match the level of advertising, xt , to demand, rt . Demand follows a random Markov process, wherein each period it may increase or decrease. Each period, demand moves up or down by k. If it increased last period, the probability of increase this period is $p \geq 1/2$, otherwise the probability of increase is $1 - p$. That is, the demand process features binary persistence. The initial level of demand is given by r0.

The firm observes the current level of demand in each period. At the beginning of time, the firm chooses x0, the initial level of advertising. At the end of any period t, the firm may choose xt ∈ R+ and pay a cost $c > 0$ to change the level of advertising to that xt . In an arbitrary period t, the firm's current level of advertising x(t) is given by the largest $\tau < t$ such that xτ 6= xτ−1. Namely, τ was the last period that the advertising was changed in, not including the current period. The firm's per period profit is given by u(t) = f(x(t), rt), where f is decreasing in the distance of its two arguments. It may be helpful to think of f as the losses from incorrect advertising, rather than the actual level of profit. Note, the firm earns profit before it can change the level of advertising. Let ti denote the ith time that the firm has changed its level of advertising, and let I be the collection of all such times. Then, firm's overall profit at time t is the discounted sum of per period utility:

$$v(t) = \sum_{t}^{T} \delta^t f(x(t), r_t) - \sum_{I}^{T} \delta^{t_i} c \qquad (1)$$

We specialize to the quadratic loss case, but conjecture that our results generalize to the case where f is any concave function:

Assumption 1.

$$f(x(t), r_t) = -(x(t) - r_t)^2 \qquad (2)$$

IV. Results

IV.I. Two Period Setup

To illustrate our main point, we focus on the case wherein a firm changes its advertising over two periods, that is, T = 2. For the firm to maximize profit by investing in the most optimal amount of advertising, portrayed by x ∗ 0 , it must initially find the most optimal level of advertisement at its starting point, or t = 0. The firm must use an arbitrary number and plug it into the expected utility function, noted by u0. Then, the firm must solve for x ∗ 0 by taking the derivative of said function and setting it equal to zero, as this would find the maximum of the function. This would give the firm the most optimal amount of advertising to start off with. The optimal strategy ensures that the firm will choose a level of advertising that will respond in the best way possible to the given circumstances. That is, the choice of the level of advertising is not restricted by demand and advertising levels. In particular, if it is optimal to increase advertising by a certain amount under a fixed demand, then it is optimal to increase it to the same relative level compared to a different demand. Thus, the firm is able to make the optimal choice without having to take into account their current levels. Accordingly, since the same relative decisions are made, we can compare the optimal utilities from two different original states of demand and advertising and find that the utilities are ultimately equal.

Lemma 1. *When the firm opts to change its level of advertising at the end of period 1, the expected utility from the next period under the optimal strategy is independent of the current levels of demand and advertising*.

Proof. To prove this, we compare the optimal utilities from two different initial states, (D, A) and (D0 , A0) where D denotes current demand and A current advertising and D0 , A0 alternate values for current demand and advertising. The utility from choosing x 0 under D, A, u(C, D, A, ↑), is the following

$$-p[D+k-x']^2-(1-p)[D-k-x']^2-C \tag{3}$$

On the other hand, the utility of $u(C, D', A', \uparrow)$ under advertising $x"$ is:

$$-p[D+k-x"]^2-(1-p)[D-k-x"]^2-C \tag{4}$$

Through setting $x"$ as follows:

$$x"=D'-D+x' \tag{5}$$

And plugging it back into $u(C, D', A', \uparrow)$:

$$\begin{aligned}-p[D+k-(D'-D+x')]^2-(1-p)[D-k-(D'-D+x')]^2-c\\ =-p[D+k-x']^2-(1-p)[D-k-x']^2-c\end{aligned}$$

The values from the two equations are rendered equal. Since the utilities hold the same value and this is true for any x 0 it must also hold for the optimal value of x 0 . Then, the optimal utilities under the two different initial states are the same.

Whether demand increased or decreased in the previous period, ultimately conditional on changing the level of advertising, the firm will receive the same level of utility in the optimal mechanism. More specifically, the expected utility remains the same if demand goes up twice in a row and the firm changes the level of advertising. To see why, suppose it were optimal to increase advertising relative to demand, conditional on demand having increased in the previous period. Then, had demand decreased instead, it would have been optimal to decrease advertising by an equivalent amount. Doing so, ensures the firm receives the same expected utility, independent of the direction of last period's demand shift.

Lemma 2. When the firm opts to change its level of advertising at the end of period 1, the expected utility from the next period under the optimal strategy is independent of whether demand goes or down in period 1.

$$-p[r+k-x_u]^2-(1-p)[r-k-x_u]^2-c \tag{6}$$

On the other hand, the utility of $u(c, x_d, \downarrow)$ is:

$$(1-p)[r+k-x_d]^2-p[r-k-x_d]^2-c \tag{7}$$

Through setting x_d as follows:

$$x_d=r-(x_u-r) \tag{8}$$

Simply put, x_d is below r to the same extent x_u was above r. The new value for x_d is then plugged back into the equation for $u(c, x_d, \downarrow)$:

$$\begin{aligned}&(1-p)[r+k-(r-[x_u-r])]^2 - p[r-k-(r-[x_u-r])]^2 - c\\&= (1-p)(-r+k+x_u)^2 - p(-r-k+x_u)^2 - c\\&= u(c, x_u, \uparrow)\end{aligned}$$

The values from the two equations are rendered equal. Since the utilities hold the same value and this is true for any x it must also hold for the optimal value of $x*$. Then, the optimal utilities under the two different initial states are the same

Lemma 3. When the firm opts to change its level of advertising at the end of period 1, the expected utility next period is always:

$$u(c) = -4k^2(1-p)p \tag{9}$$

Proof. Because of Lemmas 1 and 2, we can assume demand increased in period 1, and set the current level of demand to 0. Then, the expected utility conditional on changing advertising to x is $-p(k-x)^2 - (1-p)(-k-x)^2$. Taking the first order condition and solving for x generates, $x = -k(1 - 2p)$. Last, plugging x back into the expected utility generates equation 9.

The firm is able to calculate the overall expected utility starting from period 0 and continuing over the following two periods by accounting for anticipated changes in demand. This expected utility comprises several components. First, the utility conditional upon the change in demand in period 1. The firm anticipates it will act optimally at the end of period 1, making a decision at that point regarding whether to change the level of advertising. The firm must account for the fact that the current level of demand will impact the probability that in period 1 it chooses to change the level of advertising.

Then, the firm must calculate the utility conditional on the probability distribution over period 2 demand. Lemma 3 implies that the utility conditional on having changed the level of advertising at the end of period 1 remains the same whether demand went up or down in period 1. Then, the firm's overall expected utility is the sum of four different terms: the utility in beginning of period 1, the utility if demand goes up during period 2 given advertising levels were not changed at the end of period 1, the period 2 utility if demand decreases given advertising levels were not changed at the end of period 1, and the period 2 utility when advertising levels are changed in period 1. We compute each of these terms, then take the derivative with respect to x_0, the initial level of advertising, and use a first-order condition approach to characterize the optimal initial advertising level.

Theorem 1. The optimal firm welfare when T = 2 is:

$$\max\left\{-8[1-p]pk^2 - c, \frac{c - cp^2 + k^2p(-9+8p^2)}{1+p}, \frac{k^2(-1-6p-2p^2-8p^3+16p^4)}{2(1-p+p^2)}\right\} \tag{10}$$

Proof. As it turns out, the optimal strategy for the firm in the second period, depends on the following two thresholds. If the cost to change the level of advertising is above both thresholds, the firm never changes its advertising in the second period. If the cost is below both, it always changes its level of advertising in the second period. Last, if it is in between the two, it changes the level of advertising only if demand decreased.

$$c_1(x) = -(4[1-p]pk) + p(r_1 - 2k - x)^2 + (1-p)(r_1 - x)^2$$
$$c_2(x) = -(4[1-p]pk) + p(r_1 + 2k - x)^2 + (1-p)(r_1 - x)^2$$

These thresholds are computed by comparing the utility from changing the level of advertising to the utility from leaving them fixed and solving for c. We can then break the problem down into the above subcases, conditional upon where c falls on a case by case basis. Then, in each subcase, we proceed by determining the optimal value for x0. To begin, we calculate V_0 when $c < c_1(x)$, which implies that the level of advertising will always be changed in the second period.

$$V_0(x_0) = -p(r_1 + k - x_0)^2 - (1-p)(r_1 - k - x_0)^2 - 4[1-p]pk^2 - c$$

Next, we take the first order condition:

$$\frac{\partial V_0}{\partial x} = 2p(r_1 + k - x) + 2(1-p)(r_1 - k - x) = 0 \quad (11)$$

Solving the above equation for x implies:

$$x_0 = k(-1 + 2p) + r_1 \quad (12)$$

This is unsurprising, conditional on always changing the level of advertising in the second period, the optimal level of advertising in the first period is simply the myopic optimum, seen in Lemma 3 adjusted appropriately for current demand. Plugging this value of x_0 back into V_0 generates an expected utility of:

$$V_0(x_0) = -8[1-p]pk^2 - c \tag{13}$$

Next, we proceed by conducting the same series of calculations, except now $c_1(x) < c < c_2(x)$, that is the level of advertising is changed only if demand went down rather than up as expected.

$$V_0(x) = -p(r_1+k-x)^2 - (1-p)(r_1-k-x)^2 - p[p(r_1+2k-x)^2 + (1-p)(r_1-x)^2] - (1-p)[4(1-p)pk^2 - c]$$

Taking the first order condition implies:

$$\frac{\partial}{\partial x_0}V_0(x_0) = -p(-2(1-p)(r_1-x_0) - 2p(2k+r_1-x_0)) + 2(1-p)(-k+r_1-x_0) + 2p(k+r_1-x_0)$$

Setting equal to 0 and solving for x_0 :

$$x_0 = \frac{k(-1+2p+2p^2)}{1+p} + r_1$$

Then, plugging x_0 back into the original equation:

$$V_0(x_0) = \frac{c - cp^2 + k^2p(-9+8p^2)}{1+p}$$

Last, when $c > c_2(x_0)$, the level of advertising is never changed:

$$\begin{aligned} V_0(x_0) = {} & -p(r_1+k-x_0)^2 - (1-p)(r_1-k-x_0)^2 - p[p(r_1+2k-x_0)^2 \\ & - (1-p)(r_1-x_0)^2] + (1-p)[-p(r_1-2k-x_0)^2 - (1-p)(r_1-x_0)^2] \end{aligned}$$

Taking the first order condition implies:

$$\frac{\partial}{\partial x_0}V_0(x_0) = 0$$
$$0 = (1-p)(2(1-p)(r_1-x)+2p(-2k+r_1-x)) - p(2(1-p)(r_1-x)$$
$$-2p(2k+r_1-x)) + 2(1-p)(-k+r_1-x) + 2p(k+r_1-x)$$
$$\implies x = \frac{k(-1+4p^2)}{2(1-p+p^2))+r_1}$$

Then finally we have:

$$V_0(x_0) = \frac{k^2(-1-6p-2p^2-8p^3+16p^4)}{2(1-p+p^2)} \tag{14}$$

Ultimately, whichever of these three values is greatest is the one that will be chosen by an optimal firm. Simply put, the optimal value is the maximum over all three.

V. Naivety

Suppose a firm lacked foresight, and simply set its level of advertising based upon the current level of demand. That is, the firm ignored the persistence in demand trends. We show that such a firm does worse relative to a firm that chooses advertising optimally when the persistence in demand increases. Formally, define the na¨ıve strategy as the one where the firm sets advertising, x_t equal to demand, r_t in every period.

We proceed by computing the expected payoff from the na¨ıve strategy:

$$u(naive) = -p(r+k-r)^2 - (1-p)(r-k-r)^2 - p[p(r+2k-[r+k])^2$$
$$+(1-p)(r-[r+k])^2] - (1-p)[p(r-2k-[r-k])^2 + (1-p)(r-[r-k])^2] - 2c$$
$$= -2k^2 - 2c$$

Then, we take the derivative in respect to p.

$$\frac{\partial}{\partial p}u(naive) = -2k^2$$
$$= 0$$

Note that the optimal strategy improves with respect to p. Therefore, the na¨ıve strategy becomes relative worse as the persistence in demand increases. That is, a firm that myopically sets advertising equal to current demand does relatively worse as more information is present with respect to demand.

VI. Conclusion

We created a model in which a firm determines the optimal level of advertising in response to stochastic demand when aiming to maximize its expected profit over time. Our model shows that setting advertising levels higher than demand levels is beneficial when demand is persistent and has previously increased. In particular, a na¨ıve strategy that fails to take persistence into account does especially bad in real-world settings where stock trends are largely consistent over time.

Notably, our definitions of demand and advertising are broad. The research question we studied, could have been reframed as the optimal decision of a firm that aims to match one quantity to another stochastic but persistent quantity. For instance, we touch on the menu costs literature by providing insight when inflation isn't deterministic.

Following the optimal strategy allows a firm to maximize their profits and minimize inappropriate expenditure on advertising. Utilizing the optimal model takes into accounts factors such as demand and trends, which firms may find helpful. This is especially the case when a firm may find themselves focused on other elements such as audience, platforms, and more. Utilizing a demand and anticipation-focused model allows firms to take into account more elements than they may see at a first glance. For example, that firm could take engagement into account to improve its assessment of future demand. An extension of our model would be able to incorporate this by generalizing demand from a binary variable to a continuous distribution. We think that this could be a productive question for future research

References

[1]. Leslie Levesque, Bob Flanagan, and Mark Lauritano. Economic impact of advertising in the united states, 2015. URL https://www.ana.net/content/show/id/37679.

[2]. Caron Beesley. How to set a marketing budget that fits your business goals and provides a high return on investment, 2012. URL https://www.sba.gov/taxonomy/term/15051? page=37.

[3]. Peter A Diamond. A model of price adjustment. Journal of economic theory, 3(2):156–168, 1971.

[4]. Laurence Ball and N Gregory Mankiw. Asymmetric price adjustment and economic fluctuations. The Economic Journal, 104(423):247–261, 1994.

[5]. George A Akerlof and Janet L Yellen. A near-rational model of the business cycle, with wage and price inertia. The Quarterly Journal of Economics, 100(Supplement):823–838, 1985.

[6]. Tore Ellingsen, Richard Friberg, and John Hassler. Menu costs and asymmetric price adjustment. Mimeo, 2006.

[7]. Jakob B Madsen and Bill Z Yang. Asymmetric price adjustment in a menu-cost model. Journal of economics, 68(3):295–309, 1998.

[8]. Mark J Zbaracki, Mark Ritson, Daniel Levy, Shantanu Dutta, and Mark Bergen. Managerial and customer costs of price adjustment: direct evidence from industrial markets. Review of Economics and statistics, 86(2):514–533, 2004.