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Reinsurance Counterparty Credit Risk and Optimal Regulatory Capital under Distribution Uncertainty

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Abstract : *Due to the limited size of the insurance market, insurance companies usually purchase insurance from a few reinsurance companies with large differences. At this time, using the Vasicek model to describe the counterparty credit risk will be inaccurate; besides, the insurance company's understanding of the counterparty default threshold distribution is incomplete, which makes it difficult to effectively determine the counterparty default probability. Therefore, the assumption that the default threshold distribution obeys a uniform distribution is inaccurate. This paper focuses on the issue of reinsurance counterparty credit risk and optimal capital supervision under distribution uncertaity. Based on the method of WCVaR,the credit risk measurement model of the reinsurance counterparty is constructed; then under default threshold distribution uncertainty, the default probability estimate is given, thereby giving the corresponding optimal regulatory capital estimate; finally, simulation analysis is used to compare the revised model with the historical model to verify the rationality and validity of the model.*

Keywords - *Reinsurance, counterparty credit risk, distribution uncertainty, default probability, optimal regulatory capital*

I. Introduction

Reinsurance business is a derivative form of insurance. Reinsurance companies are also known as counterparties of a given insurance company. In the process of reinsurance, insurance companies must guard against counterparty credit risks. Therefore, studying the credit risk of the counterparty and clarifying the regulatory capital of the reinsurance counterparty has important theoretical and practical significance for promoting the long-term and healthy development of reinsurance business in our country.

As we know, due to the limited size of the insurance market, insurance companies usually purchase insurance from a few reinsurance companies with large differences. At this time, using the Vasicek model to describe the counterparty credit risk will be inaccurate; secondly, considering the actual reinsurance process, one of the prerequisites for insurance companies to measure counterparty credit risk is the known probability of default. In previous studies, the credit rating of an authoritative institution was often used as the basis for judging the company's default probability. However, since Yongmei, Brilliance, Ziguang and other AAA high-credit rating state-owned enterprise bonds defaulted one after another, "rating inaccuracy" has been frequently exposed to the public as a normal state, which has seriously affected investors' decision-making and judgment. That is to say, the insurance company's understanding of the counterparty default threshold distribution is incomplete, which makes it difficult to effectively determine the counterparty default probability. Therefore, the assumption that the default threshold distribution obeys a uniform distribution is inaccurate.

Ter Berg [1] and Sandstrom [2] proposed a common shock method, which can describe the credit risk of reinsurance counterparties using a given variety of default losses. This common shock will affect the possibility of a particular reinsurance company defaulting, and it will also affect the correlation between them. It is promoted in Hendrych et al. [3] and can be used to calculate the regulatory capital requirements for preventing credit risk to cover the credit risk in portfolios with fewer heterogeneous counterparties.

Although many well-known scholars have discussed the optimal reinsurance and investment issues, only a few scholars have considered the uncertainty of the model. In fact, the rate of return of risky assets is difficult to accurately estimate, and for risk-preferred insurance companies, model parameters and expected losses also have the same uncertainty. Therefore, some scholars have proposed and studied the influence of model uncertainty on the corresponding optimal investment portfolio problem of investors or insurance companies. The methods to solve the uncertainty of the model are now mainly robust and games (including zero-sum games and non-zero-sum games). Kang et al. [4] combined a robust data-driven method to construct a set with fuzzy mean and variance, proving that the robust investment portfolio constructed under the condition of uncertain distribution is still superior.

This paper focuses on the issue of reinsurance counterparty credit risk and optimal capital supervision under distribution uncertainty. Based on the method of WCVaR, the credit risk measurement model of the reinsurance counterparty is constructed; then under default threshold distribution uncertainty, the default probability estimate is given, thereby giving the corresponding optimal regulatory capital estimate; Finally, simulation analysis is used to compare the revised model with the historical model to verify the rationality and validity of the model.

II. Preliminaries

In the traditional Markowitz mean-variance model, risk can be quantified by the variance of loss. But for credit risk, its asymmetry is very significant, so the loss distribution is asymmetric and highly skewed, which does not satisfy the assumption that the model loss follows a normal distribution. Therefore, scholars revised it and proposed the VaR method.

VaR is the abbreviation of Value-at-Risk. In general, assuming that at the confidence level β, the probability of risk loss L does not exceed, the confidence level $\beta \in (0,1)$, VaR can be expressed as:

$$\mathrm{VaR}_\beta(x) = \inf\{\alpha \in \square : \mathrm{Pr\,ob}\{L \le \alpha\} \ge \beta\}$$

Because VaR does not necessarily satisfy subadditivity, the academic community proposed CVaR in 2000, which is called Conditional Vaule-at-Risk. It is defined as the average value of the tail distribution exceeding VaR under a certain confidence level and within the holding period:

$$\mathrm{CVaR}_\beta(x) = E[L \mid L \ge \mathrm{VaR}_\beta(x)] \tag{1}$$

Let $L(x,y)$ denote the credit risk loss, where x is the decision vector, which denotes the weight of credit assets in the financial asset portfolio; y is a random vector. Suppose that y obeys a continuous distribution, and express its density function as $p(\cdot)$, which $\mathrm{CVaR}_\beta(x)$ can be expressed as the expected loss value exceeding $\mathrm{VaR}_\beta(x)$:

$$\mathrm{CVaR}_\beta(x) = \frac{1}{1-\beta}\int_{L(x,y)\ge \mathrm{VaR}_\beta(x)} L(x,y)\,p(y)\mathrm{d}y$$

Uryasev and Rockafellar [5] showed $[t]^+ = \max\{0,t\}$, and CVaR can be calculated by minimizing the function:

$$\mathrm{CVaR}_\beta(x) = \min_{\varepsilon\in\square} F_\beta(x,\varepsilon) \tag{2}$$

Using enumerated methods, the approximate value of $F_\beta(x,\varepsilon)$ can be expressed as:

$$\tilde{F}_\beta(x,\varepsilon) = \alpha + \frac{1}{S(1-\beta)}\sum_{k=1}^{S}[L(x,y^{[k]}) - \varepsilon]^+$$

Where S represents the number of samples, and $y^{[k]}$ represents the k sample. According to the statistical law of large numbers, when the sample size S reaches infinity, the empirical mean $\tilde{F}_\beta(x,\varepsilon)$ converges to $F_\beta(x,\varepsilon)$.

Because the premise of using CVaR for risk measurement is to know the loss distribution, if there is no information about the y obedience distribution, then it is impossible to calculate the exact value of $F_\beta(x,\varepsilon)$. Assuming that only the distribution function belongs to a certain set $\square$, that is $P(\cdot)\in\square$, then for a fixed $x \in X$, the worst-case CVaR (Worst-case CVaR) can be defined as:

$$\mathrm{WCVaR}_\beta(x) = \sup_{P(\cdot)\in\square} \mathrm{CVaR}_\beta(x) \tag{3}$$

Which is :

$$\mathrm{WCVaR}_{\beta}(x) = \sup_{P(\cdot)\in\square} \min_{\varepsilon\in R} F_{\beta}(x,\varepsilon) \tag{4}$$

III. Construction of Reinsurance Company Credit Risk Measurement Modified Model Based on WCVaR Method

In this subsection, this article first gives the calculation method of the combined credit risk loss of reinsurance counterparties. Then, based on the characteristics of the loss, based on the WCVaR method, a modified model of the signal risk measurement of reinsurance counterparties is constructed.

3.1 Characteristics of distribution of credit risk loss of reinsurance counterparty portfolio

With reference to Sandstrom et al., we assume that there are i counterparties in the entire reinsurance portfolio, $i = 1,...,k$. The default process is described by LGD_i and p_i : where LGD_i is the specific loss percentage after the default of the counterparty's risk exposure, referred to as the loss given default; p_i is the default probability of the i counterparty. If the counterparty i does not default, then $p_i = 0$, vice versa $p_i \neq 0$. Loss given default rate and probability of default are two random variables that are not independent of each other.

At this time, the loss L caused by the default in the combination can be expressed as:

$$L = \sum_{i=1}^{n} LGD_i \cdot p_i \tag{5}$$

Then we can know that the mean value of loss L satisfies:

$$\begin{aligned} E(L) &= \sum_{i=1}^{n} E(LGD_i \cdot p_i) = \sum_{i=1}^{n} [E(LGD_i)\cdot E(p_i) + Cov(LGD_i, p_i)] \\ &= \sum_{i=1}^{n} [E(LGD_i)\cdot E(p_i) + \rho_i \cdot \sigma(LGD_i)\cdot \sigma(p_i)] \end{aligned} \tag{6}$$

The covariance satisfies:

$$\begin{aligned} \Sigma(L) &= \sum_{i=1}^{n}\sum_{j=1}^{n} \Lambda(LGD_i \cdot p_i, LGD_j \cdot p_j) \\ &= \sum_{i=1}^{n}\sum_{j=1}^{n} [E(LGD_i \cdot p_i, LGD_j \cdot p_j) - E(LGD_i \cdot p_i)\cdot E(LGD_j \cdot p_j)] \\ &= \sum_{i=1}^{n}\sum_{j=1}^{n} [E(LGD_i \cdot p_i, LGD_j \cdot p_j) - [E(LGD_i)\cdot E(p_i) \\ &\quad + \rho_i \cdot \sigma(LGD_i)\cdot \sigma(p_i)]\cdot[E(LGD_j)\cdot E(p_j) + \rho_j \cdot \sigma(LGD_j)\cdot \sigma(p_j)] \end{aligned} \tag{7}$$

ρ_i represents the Pearson correlation coefficient between the default loss rate and the default probability of the counterparty.

3.2 Construction of a modified model based on WCVaR under distribution uncertainty

Combined with the research of Hendrych et al., β is the confidence level, $0 < \beta < 1$, and C is the selected minimum downward threshold. Now substitute the credit risk loss function into the formula of regulatory capital $RC_{\beta}(L)$, and establish a relationship with VaR credit risk value:

$$RC_{\beta}(L) = E(L \mid L \geq C) = \sum_{i=1}^{k} E(LGD_i^{B}, PD_i^{\square} \mid L \geq VaR_{\beta}(L)) \tag{8}$$

Assuming that L obeys an uncertain distribution family $\square$, that is $L \sim D(\cdot) \in \square$, the mean and the covariance satisfies $E(L) = \bar{\mu}$ and $\Sigma(L) = \bar{\Sigma}$. If the support set of L is covers the entire space, that is $\Omega = \mathrm{R}^n$, then it can be derived $\square = \{D \in M_{+} \mid P[(L) \in \Omega] = 1, E(L) = \bar{\mu}, \Sigma(L) = \bar{\Sigma}\}$,According to formula (1) and formula (8), we obtain the formula of minimum regulatory capital $\mathrm{RC}_{\beta}(L)$:

$$\min \mathrm{RC}_{\beta}(L) = \sup_{D(\cdot)\in\square} CVaR_{\beta}(L) \tag{9}$$

Lim et al. [6] showed that if the loss distribution is mis specified, then CVaR is sensitive to it, and they also demonstrated the vulnerability associated with CVaR minimization. This vulnerability needs to be improved with a robust distribution method. Therefore, according to formula (2), the WCVaR measurement method is introduced and expressed as:

$$\mathrm{WCVaR}_{\beta}(L) = \max_{D(\cdot)\in\square} \mathrm{CVaR}_{\beta}(L) \tag{10}$$

We use $\text{W-RC}_{\beta}(L)$ to represent the optimal regulatory capital under the loss distribution uncertainty, and the revised model of the WCVaR-based reinsurance counterparty credit risk measurement $\text{WRC-}\square$ is:

$$(\text{WRC-}\square): \text{W-RC}_{\beta}(L) = \mathrm{WCVaR}_{\beta}(L) = \max_{D(\cdot)\in\square} \mathrm{CVaR}_{\beta}(L) \tag{11}$$

IV. Valuation Process Of Optimal Regulatory Capital Of Reinsurance Counterparties

This section first revises PD that the default threshold obeys a uniform distribution, and finds its estimate based on the default threshold distribution uncertainty. Secondly, revises the constant default loss rate to random LGD. Then, the asymmetry and non-linear correlation between the two random variables, PD with distribution uncertainty and the random LGD, are again considered, and their correlation is measured using the generalized correlation coefficient GMC. Finally based on the above theory, the optimal regulatory capital valuation for the credit risk of the reinsurance counterparty is studied and given.

4.1 Estimation of Reinsurance Counterparty Default Probability under the Default Threshold Distribution

In chapter 3.1, we gave a calculation method for the combined credit risk loss of reinsurance counterparties. It can be observed that the probability of default p_i is a parameter that needs to be processed based on sample data. However, in most previous literatures on credit risk of reinsurance counterparties, external credit ratings provided by specialized agencies are generally used.

In order to ensure the accuracy and usability of the loss calculation, we refer to the models of Giesecke et al.[7] and Kang et al.[4] below to modify the probability of default that the default threshold distribution obeys a uniform distribution, so that it can still be calculated under the assumption that the default threshold distribution uncertainty. In addition, we also referred to ter Berg [1] and added the common shock variable to ensure that it is still applicable in the reinsurance field.

4.1.1 PD Subject to Prior Distribution

In the real financial environment, due to future market uncertainty and asymmetry in investment information, different insurance companies have different income forecasts when making reinsurance decisions, which leads to different insurance companies' expectations for the distribution of counterparty default thresholds.

. Assuming that at zero time, the counterparty issues a discount bond with a holding time of T , then there is a possibility of default before maturity. The insurance company expects that the reinsurance counterparty will default. If k is used to represent the asset value, when $V_t < k$, the insurance company expects that the counterparty will default when the value of the enterprise's assets is k at this time. K is the default threshold. It means that the insurance company expects the logarithmic growth rate of the asset value of the reinsurance counterparty. Due to the divergence in the forecast of the income, K is not a constant. So $k = V_0 e^{K}$, k is also not a constant.

According to the hypothesis of Giesecke et al. [7] in the study, k is an unobservable continuous random variable and obeys a prior probability distribution $D(\cdot)$. The insurance company only includes the bond prices disclosed by the counterparty and the information whether there is a default, which is expressed as a right continuous filter $G_t = L_t \vee \sigma(N_s : s \le t) \in L$. G_t represents the total information filtering of the insurance company on the reinsurance counterparty bond price, asset value, default status, and income forecast divergence. Define the first arrival time τ_x as $\tau_x = \inf\{t > 0 : V_t \le x\}$ .Then there is a trans Gaussian process $P(\tau_x \le t) = P(\min_s \le V_t \le x)$. Further referring to Karatzas and Shreve [8], we have obtained the specific distribution density $f(t,x)$ of the first arrival time τ_x . We use θ to represent the maximum possible value of the company's asset value at the time of default, and substitute it into the default probability p_i of the counterparty i :

$$p_i = P[\tau \le T \mid G_t] = 1 - E[\frac{D(H_T)}{D(H_t)} \mid G_t] = 1 - \frac{1}{D(H_t)} \int_{-\infty}^{H_t \wedge \frac{\theta}{x_0}} D(y) f(T-t, y-H_t) dy \qquad (12)$$

4.1.2 The baseline PD under the default threshold distribution uncertainty

In Giesecke et al.[7], when investors in the market expect a company to default, its asset value k obeys a uniform distribution on $[0, X_0]$ and satisfies $k = V_0 e^K$. At this time, the prior distribution is $D(x) = e^x$. However, this is only an assumption for a rational market. In fact, in the real financial market, it is impossible for the enterprise value to be zero when the counterparty declares default, and it is almost impossible to obtain extreme values close to both sides of the distribution. In addition, it is impossible to be within the range of corporate asset values $[0, V_0]$. Each value has the same number of insurance companies that are expected to default. The company's asset status and investor confidence will affect the insurance company's decision-making expectations for default. The experimental distribution $D(\cdot)$ is uncertain.

This article refers to the uncertainty set of moment information defined by Delage and Ye [9], combined with the portfolio model under distribution uncertainty of Kang [4]. In this paper, the distribution uncertainty theory is applied to the field of credit risk, and the probability of default model when the counterparty default threshold distribution is uncertain.

Let μ_k denote the expected mean value of the asset value of the insurance company when the counterparty defaults, and Σ_k denote the expected covariance of the asset value. Assuming that the prior distribution $D(\cdot)$ is an uncertain distribution $\square$, the distribution only knows part of the moment information, and the set of uncertain distributions $\square(\gamma_1, \gamma_2)$ is defined as:

$$\square(\gamma_1, \gamma_2) = \begin{Bmatrix} [\mu_k - \hat{\mu}_k]^T \hat{\Sigma}_k^{-1} [\mu_k - \hat{\mu}_k] \le \gamma_1 \\ \left\| \Sigma_k - \hat{\Sigma}_k \right\| \le \gamma_2, \Sigma_k \succ 0 \end{Bmatrix} \qquad (13)$$

$k = X_0 e^K$, which represents the default threshold of the counterparty.

Given the sample mean vector $\hat{\mu}_k \in R^n$ and sample covariance matrix $\hat{\Sigma}_k \in R^{n\times n}$, the parameters γ_1 and γ_2 determine the size of the uncertainty set, which is a measure of the uncertainty of the expected return μ_k and covariance estimation Σ_k . They provide a way to quantify people's true values of γ_1 and γ_2 . Through γ_1 and γ_2 construct the set of uncertain distributions that the default threshold obeys k . The meaning of this set is to construct an ellipsoid set with and as the radius according to the known parameters. We call the positive parameters γ_1 and γ_2 the uncertainty level, which can be estimated based on the sample observations of the default threshold.

Therefore, when the default threshold distribution is uncertain, the default probability model of the counterparty i can be expressed as:

$$\max_{D(\cdot)\in\square(\hat{\mu}_k, \hat{\Sigma}_k, \gamma_1, \gamma_2)} p_i = \max_{D(\cdot)\in\square(\hat{\mu}_k, \hat{\Sigma}_k, \gamma_1, \gamma_2)} [1 - \frac{1}{D(H_t)} \int_{-\infty}^{H_t \wedge \frac{\theta}{x_0}} D(y) f(T-t, y-H_t) dy] \qquad (14)$$

When the equal sign is taken in □ (γ_1, γ_2), formula (14) obtains the maximum value, which is the maximum probability of default p_i. We consider it to be the benchmark default probability, namely:

$$p_i = 1 - \frac{1}{D(H_t)} \int_{-\infty}^{H_t \wedge \frac{\theta}{x_0}} D(y) f(T-t, y-H_t) dy] \tag{15}$$

Where $D(\cdot)$ is the set of uncertain distributions subject to k, then the mean and covariance of p_i satisfy:

$$\mu_k = \widehat{\mu}_k + \sqrt{\gamma_1 \widehat{\Sigma}} \tag{16}$$

$$\Sigma_k = \widehat{\Sigma}_k + \gamma_2 E_n \tag{17}$$

4.1.3 The modified PD added to the common shock variable

With reference to ter Berg [1] and Sandstrom [2], a good description of the credit risk of reinsurance counterparties is to consider combining a random variable called common shock. The random variable of this common shock will not only affect the probability of default of a particular reinsurance company, but also affect the correlation between reinsurance portfolios. This section will briefly describe this method and try to introduce it to modify the above-mentioned benchmark probability of default p_i based on the distribution uncertainty, so as to ensure that the modified probability of default PD_i is still applicable in the field of reinsurance.

The reinsurers of an insurance company are often impacted by the real environment, such as financial crises caused by economic recession, changes or reforms of relevant laws, and natural disasters. We call this a common impact, which is represented by a random variable S. The numerical range is $0 < S < 1$. When S rises from 0 to 1, it means that the common shock of a given event on the insurer has changed from very low to higher. The degree of change of S can be expressed by the probability density function of the following form, $f(S \mid \lambda) = \lambda S^{\lambda-1}$, $0 \le S \le 1$, $0 < \lambda < 1$.

The common shock S will determine the probability of default PD. In ter Berg [1], the functional relationship between the two is set as:

$$PD_i(s) = p_i + (1-p_i) s^{\tau/p_i}, \quad 0 < s < 1 \tag{18}$$

Among them p_i is the basic level of the default probability of the i counterparty ($i = 1, 2, ..., n$) in the reinsurance portfolio (the value after excluding the influence of the common shock S), which is the benchmark default probability under distribution uncertainty described above.

By using the probability density function $f(s \mid \alpha)$ of S to integrate the modified default probability function $PD_i(s)$, the expected value of the default probability PD_i of the i counterparty is:

$$PD_i = E(PD_i(D)) = \int_0^1 PD_i(s) f(s \mid \lambda) ds = \int_0^1 [p_i + (1-p_i) s^{\tau/p_i}] \cdot \lambda \cdot s^{\lambda-1} ds = \frac{(\lambda/\tau + 1) p_i}{1 + \lambda/\tau \cdot p_i} \tag{19}$$

Substituting equation (19) into equation (15), then $PD_i^{\square}$ based on distribution uncertainty and common shock can be shown as :

$$PD_i^{\square} = \frac{(\lambda/\tau + 1) \cdot p_i}{1 + \lambda/\tau \cdot p_i} = \frac{(\lambda/\tau + 1) \cdot [1 - \frac{1}{D(H_t)} \int_{-\infty}^{H_t \wedge \frac{\theta}{x_0}} D(y) f(T-t, y-H_t) dy]}{1 + \lambda/\tau \cdot [1 - \frac{1}{D(H_t)} \int_{-\infty}^{H_t \wedge \frac{\theta}{x_0}} D(y) f(T-t, y-H_t) dy]} \tag{20}$$

Where $0 < \lambda < 1$, $0 < \tau < 1$. The prior distribution $D(\cdot)$ is obtained according to the estimation of the uncertain set □ , and its mean and covariance satisfy equations (16) and (17).

4.2 Estimate of Random LGD and Correlation Coefficient

In Hendrych et al.[3], LGD is generally simply defined as $1/N$, without considering the possible distribution of the default loss rate. This is obviously not in line with the real environment, and it is extremely easy to cause errors in the measurement of credit risk. This paper refers to the latest research results of Moody

Company, the LGD presents a bimodal distribution. Therefore, it can be guessed that the LGD of reinsurance companies also conforms to the bimodal beta distribution.

In the past research, Pearson's correlation coefficient has been the most important correlation measure used in many studies since it began to pay attention to the correlation between random variables. But it have great limitations such as it cannot explain the asymmetry of the variance between nonlinearly correlated random variables.

Since the distribution of the probability of default $PD^{\square}$ in this paper is uncertain, and the mean and covariance of the distribution are determined to belong to an uncertain set of moment information, the distribution can be regarded as a group. However, LGD^{B} obeys the exact distribution, so the problem that needs to be solved is the asymmetry and nonlinear correlation between the uncertain distribution and the certain distribution.

This article refers to the correlation measurement method proposed by Zheng et al. [10]: Generalized Correlation Coefficient (GMC). Assuming LGD^{B} is a random variable X , $PD^{\square}$ is a random variable Y . Let ρ_G be the generalized correlation coefficient GMC, then we have:

$$\rho_G = GMC(Y \mid X) = \frac{\mathrm{var}(E(Y \mid X))}{\mathrm{var}(Y)} = 1 - \frac{E(\mathrm{var}(Y \mid X))}{\mathrm{var}(Y)} = 1 - \frac{E[(X - \mathrm{var}(Y \mid X))^2]}{\mathrm{var}(Y)} \qquad (21)$$

Chen et al. [11] gives the non-parametric estimate of ρ_G .

4.3 Valuation of optimal regulatory capital for reinsurance counterparties

According to equation (5), as well as $PD^{\square}$ and LGD^{B} , let the reinsurance counterparty portfolio credit risk loss be:

$$L = \sum_{i=1}^{n} LGD_i^{B} \cdot PD_i^{\square}$$

It can be seen from Section 4.2 that LGD^{B} obeys the bimodal beta distribution, so it is recorded as $E(LGD_i^{B}) = \mu_B$, $\sigma^2(LGD_i^{B}) = \sigma_B{}^2$; the distribution of $PD^{\square}$ is uncertain, so it is recorded as $E(PD_i^{\square}) = \mu_{\square}$, $\sigma^2(PD_i^{\square}) = \sigma_{\square}{}^2$; the correlation coefficient between LGD^{B} and $PD^{\square}$ is corrected to GMC ρ_G , then it can be seen that the mean value of L satisfies:

$$E(L) = \sum_{i=1}^{n} [E(LGD_i) \cdot E(PD_i) + \rho_i \cdot \sigma(LGD_i) \cdot \sigma(PD_i)] = \sum_{i=1}^{n} [\mu_B \cdot \mu_{\square} + \rho_G \cdot \sigma_B \cdot \sigma_{\square}] \qquad (22)$$

Let $LGD_i \cdot PD_i = x$, $LGD_j \cdot PD_j = y$, then the covariance satisfies:

$$\begin{aligned} \Sigma(L) &= \sum_{i=1}^{n}\sum_{j=1}^{n} [E(LGD_i \cdot PD_i, LGD_j \cdot PD_j) - [E(LGD_i) \cdot E(PD_i) + \rho_i \cdot \sigma(LGD_i) \\ &\quad \cdot \sigma(PD_i)] \cdot [E(LGD_j) \cdot E(PD_j) + \rho_j \cdot \sigma(LGD_j) \cdot \sigma(PD_j)] \\ &= \sum_{i=1}^{n}\sum_{j=1}^{n} [\iint x \cdot y \cdot f(x,y) dxdy - (\mu_B \cdot \mu_{\square} + \rho_G \cdot \sigma_B \cdot \sigma_{\square})^2] \\ &= \sum_{i=1}^{n}\sum_{j=1}^{n} [C - (\mu_B \cdot \mu_{\square} + \rho_G \cdot \sigma_B \cdot \sigma_{\square})^2] \end{aligned} \qquad (23)$$

$f(x,y)$ is the joint probability density between L_i of the counterparty i and L_j of the counterparty j , estimated by the Copula function, and C is the expected joint distribution of L_i and L_j .

For the convenience of analysis, we modify the form of the probability of default according to formula (20), which $\mu_{\square}$ is defined as the mean value of the probability of default, that is $E(PD_i^{\square}) = \mu_{\square}$, $\hat{\Sigma}$ is the covariance of the probability of default, $\Sigma(PD_i^{\square}) = \Sigma_{\square}$, then the probability of default also obeys the uncertain distribution family, and then define the following set:

$$U_{(\hat{\mu}_{\square}, \hat{\Sigma}_{\square})} = \{(\mu_{\square}, \Sigma_{\square}) \in \square^n \mid (\mu_{\square} - \hat{\mu}_{\square})^T \hat{\Sigma}_{\square}^{-1} (\mu_{\square} - \hat{\mu}_{\square}) \le \gamma_1, \|\Sigma_{\square} - \hat{\Sigma}_{\square}\| \le \gamma_2, \Sigma_{\square} \succ 0\}$$

Since the modified default probability $PD_i^{\square}$ obeys the uncertain distribution family , it can be known that under the condition of $(\mu_{\square}, \Sigma_{\square}) \in \square^n$, its mean and covariance satisfy:

$$(\mu_{\square} - \hat{\mu}_{\square})^T \hat{\Sigma}_{\square}^{-1}(\mu_{\square} - \hat{\mu}_{\square}) \le \gamma_1, \left\| \Sigma_{\square} - \hat{\Sigma}_{\square} \right\| \le \gamma_2, \Sigma_{\square} \succ 0$$

Which is:

$$\hat{\mu}_{\square} - \sqrt{\gamma_1 \hat{\Sigma}_{\square}} \le \mu_{\square} \le \hat{\mu}_{\square} + \sqrt{\gamma_1 \hat{\Sigma}_{\square}}, 0 < \Sigma_{\square} \le \hat{\Sigma}_{\square} + \gamma_2 E_n$$

Since the distribution of LGD^B is determined and $PD^{\square}$ is uncertain, the distribution of credit risk loss is also uncertain. Then from equation (3.16) we know that the credit risk loss function L obeys the distribution family $\square$, and $\square$ is related to $PD^{\square}$ distribution family $\square(\gamma_1, \gamma_2)$, that is $L \sim D(\cdot) \in \square(\gamma_1, \gamma_2)$, $E(L) = \bar{\mu}$, $\Sigma(L) = \bar{\Sigma}$, then the model $\text{WRC-}\square$ we give in equation (11) can be expressed as $\text{WRC-}\square(\gamma_1, \gamma_2)$, and according to above formulas, the distribution characteristics of L can be expressed as follows:

$$\sum_{i=1}^{n}[\hat{\mu}_B \cdot (\hat{\mu}_{\square} - \sqrt{\gamma_1 \hat{\Sigma}})] \le \bar{\mu} \le \sum_{i=1}^{n}[\hat{\mu}_B \cdot (\hat{\mu}_{\square} + \sqrt{\gamma_1 \hat{\Sigma}}) + \rho_G \cdot \sqrt{\hat{\Sigma}_B(\hat{\Sigma} + \gamma_2 E_n)}]$$

$$\sum_{i=1}^{n}\sum_{j=1}^{n}[C - [\hat{\mu}_B \cdot (\hat{\mu}_{\square} + \sqrt{\gamma_1 \hat{\Sigma}}) + \rho_G \cdot \sqrt{\hat{\Sigma}_B(\hat{\Sigma} + \gamma_2 E_n)}]^2] \le \bar{\Sigma} \tag{24}$$

$$\bar{\Sigma} \le \sum_{i=1}^{n}\sum_{j=1}^{n}[C - [\hat{\mu}_B \cdot (\hat{\mu}_{\square} - \sqrt{\gamma_1 \hat{\Sigma}})]^2]$$

Lemma 4.1 [12] Assuming that L is the risk loss, satisfies $E(L) = \bar{\mu}$, $\Sigma(L) = \bar{\Sigma}$, let ε be the benchmark loss and $\varepsilon \in \square$, the confidence level $0 < \beta < 1$, then the following formula holds:

$$\sup_{L \sim (\bar{\mu}, \bar{\Sigma})} E[(L - \varepsilon)_+] = \frac{-\varepsilon + \bar{\mu} + \sqrt{\bar{\Sigma} + (\varepsilon + \bar{\mu})^2}}{2} \tag{25}$$

Proposition 4.1 L is assumed to be risk loss, satisfying $L = \sum_{i=1}^{n} LGD_i^B \cdot PD_i^{\square}$, LGD^B obey the bimodal beta distribution, $E(LGD_i^B) = \mu_B$, $\sigma^2(LGD_i^B) = \sigma_B^{\ 2}$, $PD^{\square}$ obey the uncertain distribution family $\square(\gamma_1, \gamma_2)$, satisfy $E(PD_i^{\square}) = \mu_{\square}$, $\sigma^2(PD_i^{\square}) = \sigma_{\square}^{\ 2}$, so L obey the uncertain distribution family $\square(\gamma_1, \gamma_2)$, $E(L) = \bar{\mu}$, $\Sigma(L) = \bar{\Sigma}$, $C = E(L_i, L_j)$. Then based on the uncertainty of the default threshold distribution and the random default loss rate, the optimal regulatory capital valuation of the reinsurer's credit risk is:

$$\begin{aligned} \text{W-RC}_\beta(L) = {} & \frac{\beta}{1-\beta} \cdot \sum_{i=1}^{n}[\hat{\mu}_B \cdot (\hat{\mu}_{\square} + \sqrt{\gamma_1 \hat{\Sigma}}) + \rho_G \cdot \sqrt{\hat{\Sigma}_B(\hat{\Sigma} + \gamma_2 E_n)}] \\ & + \sqrt{\frac{1-\beta}{\beta}} \cdot \sum_{i=1}^{n}\sum_{j=1}^{n}\sqrt{C - [\hat{\mu}_B \cdot (\hat{\mu}_{\square} - \sqrt{\gamma_1 \hat{\Sigma}})]^2} \end{aligned} \tag{26}$$

where $1-\beta$ is the confidence level, $0 < \beta < 1$; n is the number of counterparties, $i = 1, ..., n$, $j = 1, ..., n$.

Proof: In order to facilitate analysis, we define according to formula (30):

$$U_{(\bar{\mu}, \bar{\Sigma})} = \{(\bar{\mu}, \bar{\Sigma}) \in \square^n \mid \sum_{i=1}^{n} A \le \bar{\mu} \le \sum_{i=1}^{n} B, \sum_{i=1}^{n}\sum_{j=1}^{n}(C - B^2) \le \bar{\Sigma} \le \sum_{i=1}^{n}\sum_{j=1}^{n}(C - A^2)\} \tag{27}$$

Where:

$$A = \hat{\mu}_B \cdot (\hat{\mu}_{\square} - \sqrt{\gamma_1 \hat{\Sigma}}), \ B = \hat{\mu}_B \cdot (\hat{\mu}_{\square} + \sqrt{\gamma_1 \hat{\Sigma}}) + \rho_G \cdot \sqrt{\hat{\Sigma}_B(\hat{\Sigma} + \gamma_2 E_n)}$$

For the $\text{WRC-}\square(\gamma_1, \gamma_2)$ model, we first let it take the supremum relative to $D(\cdot) \in \square$ avoid that its maximum value does not exist, and then relative to the maximum in the uncertain set $(\bar{\mu}, \bar{\Sigma})$, we get:

$$\text{W-RC}_\beta(L) = \max_{D(\cdot) \in \square(\gamma_1, \gamma_2)} \text{CVaR}_\beta(L) = \max_{(\bar{\mu}, \bar{\Sigma}) \in U_{(\bar{\mu}, \bar{\Sigma})}} \sup_{D(\cdot) \in \square} \text{CVaR}_\beta(L) \tag{28}$$

From equation (1), we know:

$$\text{CVaR}_\beta(L)=\min_{\varepsilon\in\mathbb{R}} F_\beta(L,\varepsilon)=\min_{\varepsilon\in\mathbb{R}}\{\varepsilon+\frac{1}{1-\beta}\int[L-\varepsilon]^+ p(\cdot)\text{d}y\}=\min_{\varepsilon\in\mathbb{R}}\{\varepsilon+\frac{1}{1-\beta}\text{E}[L-\varepsilon]_+\} \quad (29)$$

From Lemma 4.1 and formula (29), formula (28) can be reduced to:

$$\begin{aligned}\text{W-RC}_\beta(L)&=\max_{\bar{\mu},\bar{\Sigma}\in U_{(\hat{\mu},\hat{\Sigma})}}\ \sup_{D(\cdot)\in\mathbb{D}}\text{CVaR}_\beta(L)\\&=\max_{\bar{\mu},\bar{\Sigma}\in U_{(\hat{\mu},\hat{\Sigma})}}\ \sup_{D(\cdot)\in\mathbb{D}}\{\min_{\varepsilon\in\mathbb{R}}[\varepsilon+\frac{1}{1-\beta}\text{E}(L-\varepsilon)_+]\}\\&=\max_{\bar{\mu},\bar{\Sigma}\in U_{(\hat{\mu},\hat{\Sigma})}}\ \min_{\varepsilon\in\mathbb{R}}\{\varepsilon+\frac{1}{1-\beta}\sup_{L\sim(\bar{\mu},\bar{\Sigma})}\text{E}[(L-\varepsilon)_+]\}\\&=\max_{\bar{\mu},\bar{\Sigma}\in U_{(\hat{\mu},\hat{\Sigma})}}\ \min_{\varepsilon\in\mathbb{R}}\{\varepsilon+\frac{1}{1-\beta}\cdot\frac{-\varepsilon+\bar{\mu}+\sqrt{\bar{\Sigma}+(\varepsilon+\bar{\mu})^2}}{2}\}\\&=\max_{\bar{\mu},\bar{\Sigma}\in U_{(\hat{\mu},\hat{\Sigma})}}\ \min_{\varepsilon\in R}\{\varepsilon+\frac{1}{2(1-\beta)}[-\varepsilon+\bar{\mu}+\sqrt{\bar{\Sigma}+(\bar{\mu}+\varepsilon)^2}]\}\end{aligned} \quad (30)$$

Let:

$$h_\beta(\varepsilon)=\varepsilon+\frac{1}{2(1-\beta)}[-\varepsilon+\bar{\mu}+\sqrt{\bar{\Sigma}+(\bar{\mu}+\varepsilon)^2}]$$

Take the first-order derivation of $h_\beta(\varepsilon)$ about ε, and let $h_\beta'(\varepsilon)=0$, then at the $h_\beta(\varepsilon)$ minimum, the optimal value of ε is:

$$\varepsilon^*=\frac{2\beta-1}{2\sqrt{\beta(1-\beta)}}\cdot\sqrt{\bar{\Sigma}}-\bar{\mu}$$

Substituting in $h_\beta(\varepsilon)$ and get:

$$h_\beta(\varepsilon^*)=\sqrt{\frac{1-\beta}{\beta}}\cdot\sqrt{\bar{\Sigma}}+\frac{\beta}{1-\beta}\cdot\bar{\mu}$$

Substitute the minimum value $h_\beta(\varepsilon^*)$ of $h_\beta(\varepsilon)$ into the formula (30), then:

$$\begin{aligned}&\max_{\bar{\mu},\bar{\Sigma}\in U_{(\hat{\mu},\hat{\Sigma})}}\ \sup_{D(\cdot)\in\mathbb{D}}\text{CVaR}_\beta(L)\\&=\max_{\bar{\mu},\bar{\Sigma}\in U_{(\hat{\mu},\hat{\Sigma})}}\ \min_{\varepsilon\in R}\{\varepsilon+\frac{1}{2(1-\beta)}[-\varepsilon+\bar{\mu}+\sqrt{\bar{\Sigma}+(\bar{\mu}+\varepsilon)^2}]\}\\&=\max_{\bar{\mu},\bar{\Sigma}\in U_{(\hat{\mu},\hat{\Sigma})}}[\sqrt{\frac{1-\beta}{\beta}}\cdot\sqrt{\bar{\Sigma}}+\frac{\beta}{1-\beta}\cdot\bar{\mu}]\\&=\frac{\beta}{1-\beta}\cdot\max_{\bar{\mu}\in U_{\hat{\mu}}}\bar{\mu}+\sqrt{\frac{1-\beta}{\beta}}\cdot\max_{\bar{\Sigma}\in U_{\hat{\Sigma}}}\sqrt{\bar{\Sigma}}\end{aligned} \quad (31)$$

Among them, it can be seen from formula (33):

$$U_{\hat{\mu}}=\{\bar{\mu}\in\mathbb{R}^n\mid\sum_{i=1}^n A\le\bar{\mu}\le\sum_{i=1}^n B\},U_{\hat{\Sigma}}=\{\bar{\Sigma}\in\mathbb{R}^n\mid\sum_{i=1}^n\sum_{j=1}^n(C-B^2)\le\bar{\Sigma}\le\sum_{i=1}^n\sum_{j=1}^n(C-A^2)\} \quad (32)$$

In formula (31), since the objective function and the constraint set are independent of each other, the original optimization problem can be solved by two independent optimization problems.

From the formula (32), we know:

$$\max_{\bar{\mu}\in U_{\hat{\mu}}}\bar{\mu}=\sum_{i=1}^n B=\sum_{i=1}^n[\hat{\mu}_B\cdot(\hat{\mu}+\sqrt{\gamma_1\hat{\Sigma}})+\rho_G\cdot\sqrt{\hat{\Sigma}_B(\hat{\Sigma}+\gamma_2E_n)}]$$

$$\max_{\bar{\Sigma}\in U_{\hat{\Sigma}}}\sqrt{\bar{\Sigma}}=\sum_{i=1}^n\sum_{j=1}^n\sqrt{C-A^2}=\sum_{i=1}^n\sum_{j=1}^n\sqrt{C-[\hat{\mu}_B\cdot(\hat{\mu}-\sqrt{\gamma_1\hat{\Sigma}})]^2}$$

Then the formula (31) can be transformed into:

$$
\begin{aligned}
&\max_{\bar{\mu},\bar{\Sigma}\in U_{(\hat{\mu},\hat{\Sigma})}} \sup_{D(\cdot)\in\square} \mathrm{CVaR}_{\beta}(L) \\
&= \frac{\beta}{1-\beta}\cdot\max_{\bar{\mu}\in U_{\hat{\mu}}}\bar{\mu} + \sqrt{\frac{1-\beta}{\beta}}\cdot\max_{\bar{\Sigma}\in U_{\hat{\Sigma}}^{\square}}\sqrt{\bar{\Sigma}} \\
&= \frac{\beta}{1-\beta}\cdot\sum_{i=1}^{n}[\hat{\mu}_B\cdot(\hat{\mu}_{\square}+\sqrt{\gamma_1\hat{\Sigma}})+\rho_G\cdot\sqrt{\hat{\Sigma}_B(\hat{\Sigma}+\gamma_2 E_n)}] \\
&+\sqrt{\frac{1-\beta}{\beta}}\cdot\sum_{i=1}^{n}\sum_{j=1}^{n}\sqrt{C-[\hat{\mu}_B\cdot(\hat{\mu}_{\square}-\sqrt{\gamma_1\hat{\Sigma}})]^2}
\end{aligned} \tag{33}
$$

The formula (28) is:

$$
\begin{aligned}
\text{W-RC}_{\beta}(L) &= \max_{(\bar{\mu},\bar{\Sigma})\in U_{(\hat{\mu},\hat{\Sigma})}} \sup_{D(\cdot)\in\square} \mathrm{CVaR}_{\beta}(L) \\
&= \frac{\beta}{1-\beta}\cdot\sum_{i=1}^{n}[\hat{\mu}_B\cdot(\hat{\mu}_{\square}+\sqrt{\gamma_1\hat{\Sigma}})+\rho_G\cdot\sqrt{\hat{\Sigma}_B(\hat{\Sigma}+\gamma_2 E_n)}] \\
&+\sqrt{\frac{1-\beta}{\beta}}\cdot\sum_{i=1}^{n}\sum_{j=1}^{n}\sqrt{C-[\hat{\mu}_B\cdot(\hat{\mu}_{\square}-\sqrt{\gamma_1\hat{\Sigma}})]^2}
\end{aligned} \tag{34}
$$

The certificate is complete.

Remark 4.1 (1) The uncertainty level γ_1 and γ_2 are estimated by the Bootstrapping algorithm through sample data, and then the distribution characteristics of the uncertainty distribution family that $PD_{\square}$ obeys are obtained; (2) Through MLE of the bimodal beta distribution parameters of LGD, the probability density function obtained is substituted into the sample data, and the specific values of the sample mean $\hat{\mu}_B$ and covariance $\hat{\Sigma}_B$ can be obtained; (3) The generalized correlation coefficient GMC can be obtained by smoothing the kernel density after processing the sample data.

V. Simulation analysis

In this section, we will use the Matlab program simulation, referring to the numerical simulation of Hürlimann (2008), mainly for the three cases of Vasicek limit probability distribution, the default threshold obeys a uniform distribution, and the default threshold distribution is uncertain. When analyzing and calculating the optimal regulatory capital, refer to Hendrych (2019), we order $\tau/\alpha=4$. This article mainly focuses on the reinsurance portfolio of N counterparties, where N is a positive integer and $N\le 20$, and the confidence level $1-\beta$ is 0.95.

Assuming the company's initial asset value following $V_0\in[60,100]$ and current t time company's value following $V_t\in[40,120]$, then we can obtain that the company's asset value process W_t at the t time, maturity time $T=1$, short-term liabilities $SD\in[20,60]$, long-term liabilities $TD\in[20,80]$, default threshold θ=90, and annualized stock fluctuations $\sigma_{\omega}{}^2\in[0.2,0.6]$ in the last two weeks. The uncertainty intensity is always 1. At this time, the mean $\mu_{\square}$ and variance $\sigma_{\square}{}^2$ of the counterparty sample distribution can be obtained to obtain the default probability of each counterparty. At the same time, the bimodal beta distribution is used to simulate the default loss rate of the counterparty

.Due to space limitations, the default probability and random loss given default rate when the distribution of default thresholds of all counterparties are uncertain are not listed. Some data are shown in table 1:

Table 1 Counterparty's parameters and probability of default

Index \ Number of counterparties	1	2	3	4	5	…	20
V_0	65.91	68.01	65.55	67.76	66.87	…	62.31
V_t	58.49	71.02	95.18	108.12	78.30	…	48.42
SD	29.33	42.59	40.98	20.52	21.77	…	34.15
TD	26.55	46.81	27.34	20.81	20.69	…	43.38
σ_w^2	0.32	0.33	0.25	0.49	0.55	…	0.57
μ_Q	72.61	66.00	54.65	30.93	32.12	…	55.84
Σq	385.50	425.72	395.69	579.59	631.40	…	652.45
PD	0.1675	0.0971	0.0704	0.2586	0.2824	…	0.3044
LGD	0.2733	0.0776	0.1299	0.8480	0.7626		0.8826

The number of samples of PD in the sample is 20, of which the maximum is 0.3063, the minimum is 0.0132, and the average is 0.1561; the number of samples of LGD is 20, of which the maximum is 0.8826 and the minimum is 0.0572. The average value is 0.4316. It can be seen that the probability of default and loss given default rate are positively correlated in the sample.

In order to compare the different performances of counterparties with different default probabilities, this paper ranks the default probabilities of each counterparty in the subsequent simulations, and uses three methods to gradually add each counterparty to the reinsurance portfolio. These three methods are the probability of default from large to small, the probability of default from small to large, and the probability of default is randomly added.

For example, the first method refers to: when there is only one counterparty in the combination, the default probability of the counterparty is 0.3063, which is the largest among the 20 counterparties. When there are two counterparties in the combination, it will be added to the ranking second. The counterparty with a probability of default of 0.2844, and so on, finally formed a reinsurance portfolio with 20 counterparties.

(1) Case 1

The counterparties are added to the portfolio by using the probability of default in descending order. Under the common shock and random default loss rate, the optimal regulatory capital of the reinsurance counterparty under the situation of uncertain default threshold distribution can be obtained, compared with the credit risk measurement model that obeys the Vasicek probability distribution and the credit risk measurement model that the default threshold distribution under the common shock obeys the normal distribution:

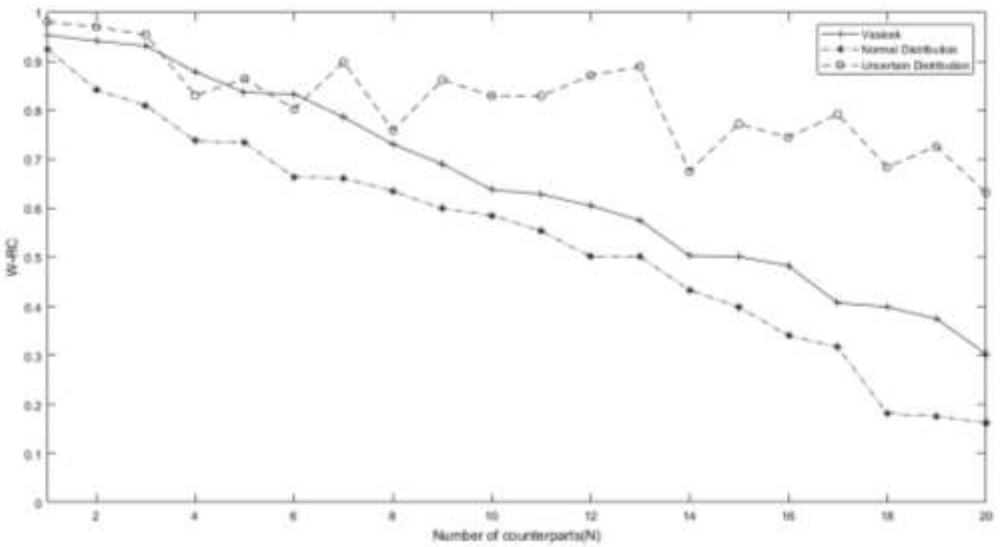

Figure 1 Under case 1 the optimal regulatory capital of the reinsurance portfolio varies with the number of counterparties in each model

As Fig.1 shows, the optimal regulatory capital measurement model with default threshold distribution uncertainty under common shocks constructed in this paper works well, and it is compatible with the credit risk measurement model that obeys the Vasicek probability distribution and the default threshold distribution under common shocks that obey the normal distribution. Compared with the credit risk measurement model, the optimal regulatory capital can better cover the largest tail loss in the reinsurance portfolio. In addition, it can be seen from the image of the model constructed in this article that when there are fewer counterparties, the value of the optimal regulatory capital basically fluctuates around 0.8, and when there are more counterparties, it will show a downward trend, so this also shows that the model is more suitable for measuring reinsurance portfolios with fewer counterparties.

(2) Case 2

Use the approach of increasing the probability of default to add counterparties to the portfolio. Under the common shock and random default loss rate, the optimal regulatory capital of the reinsurance counterparty when the default threshold distribution is uncertain is obtained, and compared with the historical model:

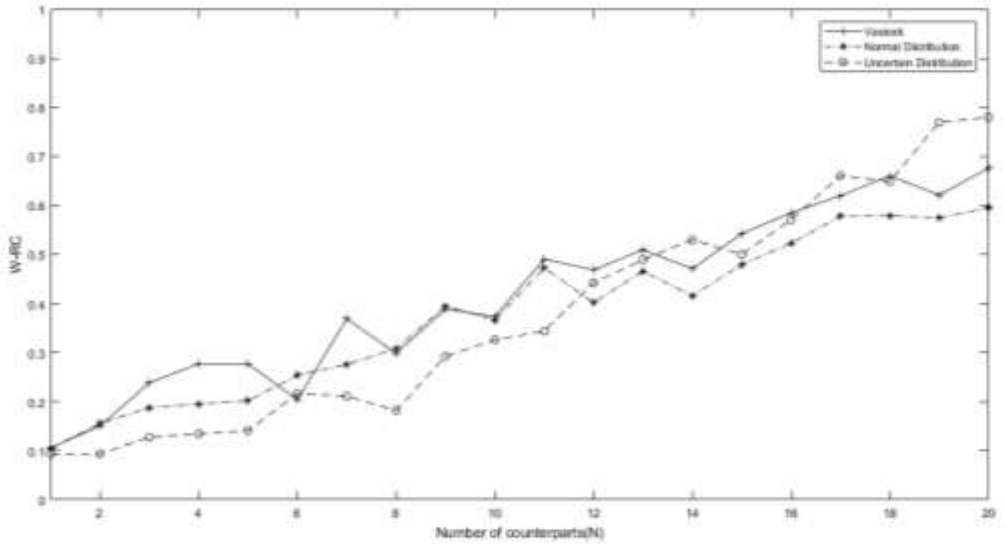

Figure 2：Under case 2 the optimal regulatory capital of the reinsurance portfolio varies with the number of counterparties in each model

As Fig.2 shows, the optimal regulatory capital measurement model with uncertain default threshold distribution under common shocks constructed in this paper is less effective than the other two historical models, and fluctuates the most. The guess may be due to the higher probability of default. The addition of counterparties makes the risk of the model rise faster, so the optimal regulatory capital increases significantly. When analyzing scenarios with fewer counterparties, the model constructed in this paper requires the lowest optimal regulatory capital, which proves that when the counterparty's default probability is low, the model can well control the credit risk of reinsurance counterparties.

(3) Case 3

Finally, the counterparty is added to the portfolio using the random probability of default method. Under the common shock and random default loss rate, the optimal regulatory capital of the reinsurance counterparty when the default threshold distribution is uncertain is obtained, and compared with the historical model.

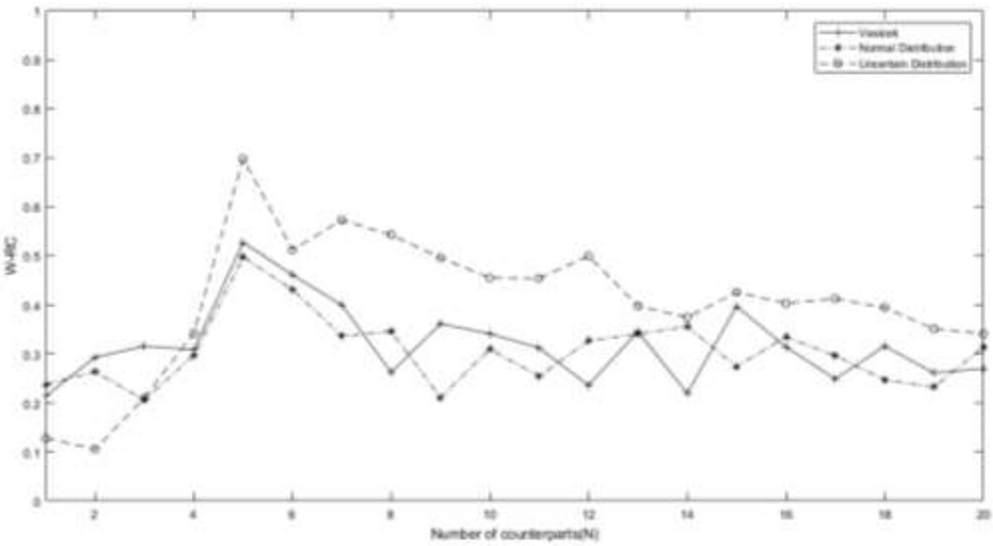

Figure 3：Under case 3 the optimal regulatory capital of the reinsurance portfolio varies with the number of counterparties in each model

As Fig.3 shows, the optimal regulatory capital measurement model with uncertain default threshold distribution under common shocks is similar to scenario 2 when the counterparty is randomly added. The optimal regulatory capital is lower when there are fewer counterparties, and when there is default When a counterparty with a higher probability joins, it will rise sharply and then decline to gradually stabilize, and show a downward trend. In this process, the optimal regulatory capital is higher than the other two historical models. This also shows that the model can well control the credit risk in the counterparty reinsurance portfolio and cover the maximum tail loss in the reinsurance portfolio when there are fewer counterparties.

VI. Conclusion

Reinsurance is a derivative form of insurance. Since reinsurance is generally a risk with high volatility and over-insurance, the insurance business contracted by insurance companies exceeds the risk range that they can bear. Risks are sub-insured to transfer part of the risks, and the insurance company that takes over is the reinsurance company. These reinsurance companies can be referred to as counterparties of a given insurance company. In the process of reinsurance, insurance companies must guard against counterparty credit risks. Counterparty credit risk is a type of credit risk, which refers to the risk that the counterparty does not pay according to the contract requirements before the final cash flow payment stipulated in the contract, and then breaches the contract.

As a new regulatory risk stipulated by formal regulations in Europe in recent years, and cases of credit risk have emerged in my country recently, this article has carried out effective risk management and accurate measurement on it. This paper focuses on the issue of reinsurance counterparty credit risk and optimal capital supervision under distribution uncertainty. Based on the method of WCVaR, the credit risk measurement model of the reinsurance counterparty is constructed; then under default threshold distribution uncertainty, the default probability estimate is given, thereby giving the corresponding optimal regulatory capital estimate; Finally, simulation analysis is used to compare the revised model with the historical model to verify the rationality and validity of the model.

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