International Journal of Business Marketing and Management (IJBMM)
Volume 5 Issue 3 March 2020, P.P. 37-46
ISSN: 2456-4559
www.ijbmm.com

The Equity Premium Puzzle undera General Utility Function

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Abstract:*In this paper, we consider the equity premium puzzle under a general utility function. We derive that the optimal strategy under a general utility function approximate the optimal strategy under the special utility function. This result posed in the present paper can be regarded as a generalization of the work by Gong and Zou [13]*

JEL classification numbers: *D9;G1;E1*
Keywords:*The equity premium puzzle; Asset pricing; The spirit of capitalism; Stochastic growth model*.

I. Introduction

The equity premium puzzle was first put forward by MrMehra and Prescott [1] in 1985, through an analysis of the American historical data over the past more than a century, they found that the return rate of stocks is 7.9%, and the return rate of the corresponding risk-free securities is only 1%, the premium is 6.9%.

Since Mr. Mehra and Prescott proposed the puzzle of equity premium in 1985, this puzzle has attracted a lot of scholars' attention. Equity premium is the compensation for stock investors to bear stock risks. The mystery of equity premium reveals a contradiction, that is, there is a big gap between the traditional asset pricing theory and the actual return and price of assets, so it is of great significance to study equity premium. Measuring and explaining equity premium is one of the most challenging research work in asset pricing theory of modern financial economics. The equity premium puzzle was usually explained by finding the inconsistencies with realities from the theoretical model, or by finding the causes and solutions to the equity premium puzzle from the empirical aspects. We mainly focus on the first aspect to study the equity premium puzzle.

Benartzi and Thaler [2], Barberis, Huang and Santos [3] explained the equity premium puzzle with the prospect theory; while Camerer and Weber [4], Maenhout [5] explained it by the Ellsberg Paradox theory. What's more, Constantinides, Donaldson and Mehra [6] studied the asset pricing with limitations on borrowing to explain the equity premium puzzle. McGrattan and Prescott [7] explained the equity premium by the change of individual income tax rate. Heaton and Lucas [8, 9] researched the equity premium based on the infinite-horizon model. Dai and Li [10] explained the equity premium under the consideration of investors' general risk aversion. Brad Barber and Odean [11] thought that the equity premium puzzle by virtue of the return rate of stocks under the markets with various costs. Rietz [12] explained the equity premium by introducing the small probability events which caused a decrease in consumption. Using the stochastic optimal control theory, Gong and Zou [13] studied the equity premium puzzle based on the domestic output process satisfying a geometric brownian motion, and derived the explicit solutions to the growth rates of consumption and savings and equilibrium on all assets under the original special utility function.

Most of the classic equity premium is based on a specific utility function, which gives a reasonable explanation of equity premium through the optimal strategy conversion in the sense of equilibrium. A natural question is whether we can also study the equity premium by analogy under the general utility function?

Based on this consideration, this paper extends the utility function to the general utility function to study the equity premium problem. Using the stochastic optimal control theory and the approach theory, this paper studies and gives a reasonable explanation of equity premium.

In this paper, we first study an investment strategy problem based on the domestic output process satisfying geometric Brownian motion, and solve the optimal investment strategy of investors in the sense of equilibrium market. It is proved that the optimal strategy under the general utility function can approach the optimal strategy under the special utility function;Secondly, with the help of the above optimal strategy, this paper calculates the expected return level of financial assets, and further obtains the corresponding premium level. Therefore, we can get the corresponding equity premium level by tracking the optimal strategy of investors. In other words, we prove that the level of equity premium under general utility function approaches to that under special utility.Finally, this paper gives an example of Equity Premium Analysis and sensitivity analysis. It is proved that the existence of the spirit of capitalism leads to the difference between the return rate of risk-free assets and that of risk-free assets. After that, this paper uses the established theoretical model to estimate the premium level of China's A-share market.

II. Equity Premium Puzzle Model

Consider a financial market consisting of two assets:the government bond B, and the capital stock K. The representative agent's wealth $W_t$has a representation

$$W(t) = B(t) + K(t).$$

Define

$$n_B = \frac{B(t)}{W(t)}, n_K = \frac{K(t)}{W(t)},$$

then we have $n_B + n_K = 1$, where $n_B$and $n_K$show the proportion of the wealth invested on the bond and the capital

We next assume that the output Y and the government expenditure G are modelled by stochastic differential equations as follows [14, 15]:

$$dY = \alpha K dt + \alpha K dy, \qquad (2.1)$$

$$dG = g\alpha K dt + \alpha K dz, \qquad (2.2)$$

whereα is the marginal physical product of the capital stock K, dy is temporally independent, normally distributed, and satisfies

$$Edy = 0, Var(dy) = \sigma_y^2;$$

besides, g is the percentage of government expenditure accounting for output,$dz$is temporally independent, normally distributed, and satisfies

$$Edz - 0, Var(dz) - \sigma_z^2.$$

If the inflation rate is stochastic, then the return on the government bond B will also satisfy a stochastic process. In the period of time dt, the stochastic real rate of the return on the bond B, dR_B is assumed to be

$$dR_B = r_B dt + du_B. \qquad (2.3)$$

where the macroeconomic equilibrium determines $r_B$and du_Bendogenously.

As for the second asset, the capital stock K, with Equation (2.1), the stochastic real rate of return on capital can be given by

$$dR_K = \frac{dY}{K} = \alpha dt + \alpha dy = r_K dt + du_K. \qquad (2.4)$$

Without loss of generality, the taxes are levied according to the capital income and the consumption c, i.e.
$dT = (\tau r_K K + \tau_c c)dt + \tau' K du_K = (\tau \alpha K + \tau_c c_t)dt + \tau' \alpha K dy$ (2.5)
whereτ, τ are the tax rates on the deterministic component and the stochastic component of the capital income, respectively, and $\tau_c$is the tax rate on the consumption.

In the absence of lump-sum taxation, government budget constraint can be described as

$$dB = BdR_B + dG - dT$$

A balanced product market requires

$$dK = dY - cdt - dG, \tag{2.6}$$

Substituting (2.1),(2.2),(2.3) and (2.5) into (2.6), we have

$$\frac{dK}{K} = [\alpha(1-g) - \frac{c}{n_K W}]dt + \alpha(dy - dz) = \phi dt + \alpha(dy - dz), \tag{2.7}$$

With the assumptions above, the representative agent's goal is to maximize his expected utility subject to the budget constraint by choosing the ratio of consumption to wealth, $\frac{c}{W}$, and the proportions of investment, $n_B$and n_K. Then the problem can be described as

$$maxE \int_0^{\infty} u\,(c_t, W_t)e^{-\beta t}dt$$

s.t.

$$dW_t = (n_B W_t r_B + n_K W_t(1-\tau)r_K - (1+\tau_c)c_t)dt + W_t d\omega. \tag{2.8}$$

$$n_B + n_K = 1 \tag{2.9}$$

Where $\beta$is the time discount rate, $u(c, W)$ is the utility function with consumption c, $d\omega = n_B du_B + n_K(1-\tau')du_K$.

This chapter mainly establishes and solves a domestic production process based on geometric Brownian motion.The optimal strategy of portfolio.First of all, according to the government's income and expenditure balance.In this way, the equilibrium market investment group is obtained question. Then, according to the principle of dynamic programming,the problem of portfolio optimization is solved in the sense of equilibrium market.The investment strategies of investors and a series of dynamic descriptions of the equilibrium system of the market are obtained.

III. Optimal solutions to the stochastic optimal control problem

To solve the agent's optimization problem above, we define the value function

$$\begin{cases} V(W(t), t) = maxE_t \int_0^{+\infty} u(c_t, W_t)e^{-\beta t}dt \\ s.t.\ (2.8), (2.9) \end{cases}$$

By the dynamic programming maximal principle, we can get the following Bellman equation

$$u(c, W) + V_t(W, t) + (\rho - (1+\tau_c)\frac{c}{W})WV_W + \frac{1}{2}\sigma_\omega^2 W^2 V_{WW} = 0 \tag{3.1}$$

where $\rho = n_B r_B + n_K(1-\tau)r_K$, and it is the expected after-tax return on total property. Taking differentiation of the Bellman equation with respect to c, $n_B$and $n_K$respectively, then we have

Proposition 3.1. The first-order conditions for the optimization problem can be written as follows

$$\frac{\partial u(c,W)}{c} = (1+\tau_c)V_W, \tag{3.2}$$

$$(r_B V_W W - \eta)dt + cov(d\omega, du_B)V_{WW}W^2 = 0, \tag{3.3}$$

$$((1-\tau)r_K V_W W - \eta)dt + cov(d\omega, (1-\tau')du_K)V_{WW}W^2 = 0, \tag{3.4}$$

$$n_B + n_K = 1, \tag{3.5}$$

whereη is the Lagrangian multiplier with the portfolio selection constraint (2.9).

Furthermore, with equation (2.7), we have

Proposition 3.2. The equilibrium system of the economy can be summarized as

$$\frac{dK}{K}=[\alpha(1-g)-\frac{c}{n_K W}]dt+\alpha(dy-dz)=\phi dt+\alpha(dy-dz)$$

$$\frac{\partial u(c,W)}{c}=(1+\tau_c)V_W$$

$$(r_B V_W W-\eta)dt+cov(d\omega,du_B)V_{WW}W^2=0$$

$$((1-\tau)r_K V_W W-\eta)dt+cov(d\omega,(1-\tau^{'})du_K)V_{WW}W^2=0$$

$$n_B+n_K=1$$

Proposition 3.3. The normal fluctuation component of the stochastic return of bonds, du_B, and the total wealth, dw, are decided by the following formulas

$$du_B=\frac{\alpha}{n_B}[(1-n_K(1-\tau^{'}))dy-dz] \quad (3.6)$$

$$d\omega=\alpha(dy-dz) \quad (3.7)$$

*Proof.*According to the inter-temporal invariance of portfolio shares [17], we have

$$\frac{dW}{W}=\frac{dK}{K}=\frac{dB}{B} \quad (3.8)$$

i.e., all the real assets grow at a common stochastic rate. Combining with (2.7), (2.8), (3.6) and (3.8), we get

$$d\omega=n_B du_B+n_K(1-\tau^{'})\alpha dy=\alpha(dy-dz)$$
$$=\frac{1}{n_B}[n_B du_B+\alpha n_K(dz-\tau^{'}dy)].$$

Noticing the fact $n_B+n_K=1$, we will get

$$du_B=\frac{\alpha}{n_B}[n_B(dy-dz)-n_K(dz-\tau^{'}dy)]$$
$$=\frac{\alpha}{n_B}[(1-n_K)dy-(1-n_K)dz-n_K dz+n_K\tau^{'}dy]$$
$$=\frac{\alpha}{n_B}[(1-n_K(1-\tau^{'}))dy-dz]$$

This ends the Proof of Proposition 3.3. □

These two equations enable us to compute all the necessary covariances and variances in the full equilibrium system.

With Proposition 3.3, we have

$$\sigma_\omega^2=\alpha^2(\sigma_y^2+\sigma_z^2)dt,$$

$$cov(d\omega,du_B)=\frac{\alpha^2}{n_B}[(1-n_K(1-\tau^{'}))\sigma_y^2+\sigma_z^2]dt,$$

$$cov(d\omega,(1-\tau^{'})du_K)=\alpha^2(1-\tau^{'})\sigma_y^2 dt,$$

Noticing equations (3.3) and (3.4), we can get

Proposition 3.4. The mean return on bonds is

$$r_B=\alpha(1-\tau)-\frac{\alpha^2 W V_{WW}}{n_B V_W}[\tau^{'}\sigma_y^2+\sigma_z^2] \quad (3.9)$$

and the stochastic growth rate of the economy is

$$\phi = \frac{r_B n_B + (g-\tau)\alpha n_K + \tau_c(c/W)}{n_B} = \rho - (1+\tau_c)\frac{c}{W} \tag{3.10}$$

where $\rho = n_B r_B + n_K(1-\tau)r_K$.

With Proposition 3.4, we now have our main theorem of this section

Theorem 3.5. The explicit solutions of the economic system are

$$n_K = \frac{c/W}{\alpha(\tau-g) + (1+\tau_c)c/W + \frac{\alpha^2 W V_{WW}}{V_W}[\tau'\sigma_y^2 + \sigma_z^2]} \tag{3.11}$$

$$n_B = 1 - n_K. \tag{3.12}$$

*Proof.*Notice the conditions

$$n_B + n_K = 1,$$
$$\rho = n_B r_B + n_K(1-\tau)r_K,$$
$$r_B = \alpha(1-\tau) - \frac{\alpha^2 W V_{WW}}{n_B V_W}[\tau'\sigma_y^2 + \sigma_z^2].$$

Combining Equations (3.9) and (3.10) to solve n_K, we can easily get (3.11). □

Next we specify the utility function as in Bakshi and Chen [18]

$$u(c, W) = \frac{c^{1-\gamma}}{1-\gamma}W^{-\lambda} \tag{3.13}$$

where$\gamma > 0$, and $\gamma \geq 1$ when $\lambda \geq 0$, and $\lambda < 0$ otherwise.

Under the form of the utility function in (3.13), we have

Proposition 3.6. The first-order optimal conditions are

$$\frac{c}{W} = \frac{\beta + \frac{1}{2}\sigma_\omega^2(1-\gamma-\lambda)(\gamma+\lambda) - \rho(1-\gamma-\lambda)}{(\gamma+\tau_c)(1-\gamma-\lambda)/(1-\gamma)} \tag{3.14}$$

$$\left(r_B - \frac{\eta}{\delta(1-\gamma-\lambda)W^{1-\gamma-\lambda}}\right)dt = (\gamma+\lambda)cov(d\omega, du_B) \tag{3.15}$$

$$\left(r_K(1-\tau) - \frac{\eta}{\delta(1-\gamma-\lambda)W^{1-\gamma-\lambda}}\right)dt = (\gamma+\lambda)(1-\tau')cov(d\omega, du_K) \tag{3.16}$$

and Equation (3.5). whereη is the Lagrangian multiplier with the portfolio selection constraint (2.9), and

$$\rho = n_B r_B + n_K(1-\tau)r_K, \tag{3.17}$$

$$\omega = n_B du_B + n_K(1-\tau')du_K, \tag{3.18}$$

$$\sigma_\omega^2 = n_B^2\sigma_B^2 + n_K^2(1-\tau')^2\sigma_K^2 + 2n_B + n_K(1-\tau')\sigma_{BK}, \tag{3.19}$$

With Proposition 3.3, (3.15) and (3.16), we have

Proposition 3.7. Under the form of the utility function in (3.14), the mean return on bonds and the stochastic growth rate of the economy are

$$r_B = \alpha(1-\tau) - \frac{\gamma+\lambda}{n_B}\alpha^2[\tau'\sigma_y^2 + \sigma_z^2] \tag{3.20}$$

$$\phi = \frac{r_B n_B + (g-\tau)\alpha n_K + \tau_c(c/W)}{n_B} = \rho - (1+\tau_c)\frac{c}{W} \tag{3.21}$$

Similar to Theorem 3.5, we have

Theorem 3.8. Under the form of the utility function in (3.14), the explicit solutions of the economic system are

$$\frac{c}{W} = \frac{\beta}{(\gamma+\tau_c)(1-\gamma-\lambda)/(1-\gamma)} - \frac{\alpha(1-\tau)+\frac{1}{2}(\gamma+\lambda)\alpha^2((2\tau'-1)\sigma_y^2+\sigma_z^2)}{(\gamma+\tau_c)/(1-\gamma)} \tag{3.22}$$

$$n_K^* = \frac{c/W}{\alpha(\tau-g)+(1+\tau_c)c/W-\alpha^2(\gamma+\lambda)[\tau'\sigma_y^2+\sigma_z^2]} \tag{3.23}$$

$$n_B^* = 1 - n_K^*. \tag{3.24}$$

wheren_K^* and $n_B^*$are the optimal proportions of investment under the utility function (3.13).

IV. Approximation of strategy under general utility

As we discussed in the previous sections, for the equity premium puzzle with utility function $u(c,W) = \frac{c^{1-\gamma}}{1-\gamma}W^{-\lambda}$, the optimal strategy is to invest a constant proportion of wealth in the risk asset. It is in general difficult to find the optimal strategy for a general utility, however, if the utility displays the behavior of a power utility when the level of wealth is very high, then the following Merton strategy can still approximately achieve the optimal value, irrespective of the initial wealth level, as long as the investment horizon is sufficient long, i.e. $t \to \infty$.

Assume that the investor consumes at a fixed proportion of wealth if the investment horizon is sufficient long, that is, $\lim_{t\to\infty}\frac{c}{W} = k$, where $0 < k < 1$ is defined as (3.22). We therefore need to estimate the difference between the optimal strategy under special utility function $n_K$and the optimal strategy under general utility function n_K^*, that is

$$|n_K - n_K^*| = \frac{\alpha^2 k(\tau'\sigma_y^2+\sigma_z^2)[-(\gamma+\lambda)-\frac{WV_{WW}}{V_W}]}{A(W)} \tag{4.1}$$

where $A(W) = [\alpha(\tau-g)+(1+\tau_c)k+\frac{\alpha^2 WV_{WW}}{V_W}(\tau'\sigma_y^2+\sigma_z^2)][\alpha(\tau-g)+(1+\tau_c)k-\alpha^2(\gamma+\lambda)(\tau'\sigma_y^2+\sigma_z^2)] > 0$

It's easy to deduce that if $\lim_{t\to\infty}|WV_{WW}+(\gamma+\lambda)V_W| \to 0$, then we have $\lim_{t\to\infty}|n_K - n_K^*| \to 0$.

The next result is referred to Bian and Zheng [18]

Lemma 4.1. Let $w \in C^{1,2}(R_+ \times R)$ be a solution to equation

$$\frac{\partial w}{\partial t} - a^2 w_{xx} = 0, w(0,x) = \phi(x) \tag{4.2}$$

Let $\psi(x) = e^{\alpha x}\phi(x)$, with constant $\alpha > 0$. Assume that $\psi \in C^1(R)$, and

$$\lim_{x\to-\infty}\frac{\psi'(x)}{e^{qx}} = -1, |\psi'(x)| \le \begin{cases} Ke^{qx}, & x \le 0 \\ K, & x \ge 0 \end{cases} \tag{4.3}$$

for some constants $K \ge 1$ and $q < 1$. Then we have, for $x \ge 0$,

$$|(e^{\alpha x}w(t,x))_x| \le KL_0(t), |(e^{\alpha x}w(t,x))_{xx}| \le K(|q|+\frac{1}{a\sqrt{\pi t}})L_0(t) \tag{4.4}$$

Where$L_0(t) = e^{\alpha^2 a^2 t} + e^{(\alpha-q)^2 a^2 t}$. Furthermore, we have

$$\lim_{x\to-\infty}\frac{(e^{\alpha x}w(t,x))_x}{e^{(\alpha-q)^2a^2t+qx}} = -1, \quad \lim_{x\to-\infty}\frac{(e^{\alpha x}w(t,x))_{xx}}{e^{(\alpha-q)^2a^2t+qx}} = -q. \tag{4.5}$$

where the convergence is uniform for $t \in [t_0, t_1]$ with any $0 < t_0 < t_1$.

Let $w(t,x) = e^{-(\alpha x + \beta t)} v(t, e^{-x})$. Then w is a solution to (4.2) with the initial condition $w(0,x) = \phi(x) = e^{-\alpha x} v(e^{-x})$, thus we have

Lemma 4.2. Assume that $u \in C^1(R)$ and satisfies

$$\lim_{y\to\infty} \frac{u'(y)}{y^{-\gamma-\lambda}} = 1, |yu'(y)| \leq \begin{cases} Ke^{1-\gamma-\lambda}, & y \leq 1 \\ K, & y \geq 1 \end{cases} \quad (4.6)$$

Then we have that for $y \geq 1$,

$$|yV_y(t,y)| \leq Ke^{\beta_0 t} L_0(t), |y^2 V_{yy}(t,y)| \leq K(1 + |1-\gamma-\lambda| + \frac{1}{a\sqrt{\pi t}}) e^{\beta_0 t} L_0(t), \quad (4.7)$$

Furthermore

$$\lim_{y\to\infty} \frac{V_y(t,y)}{e^{\xi t} y^{-\gamma-\lambda}} = 1, \lim_{y\to\infty} \frac{V_{yy}(t,y)}{e^{\xi t} y^{-(\gamma+\lambda)-1}} = -\gamma - \lambda, \quad (4.8)$$

where the convergence is uniform f or $t \in [t_0, t_1]$ with any $0 < t_0 < t_1$.

The next result shows the approximation of strategy under general utility.

Theorem 4.3. Assume that $u \in C^1(R)$, $\lim \frac{c}{W} = k$ and

$$\lim_{W\to\infty} \frac{u'(W)}{W^{-\gamma-\lambda}} = C, \ |Wu'(W)| \leq \begin{cases} Ke^{1-\gamma-\lambda}, & W \leq 1 \\ K, & W \geq 1 \end{cases} \quad (4.9)$$

holds, where $C \in R$. Then we have, for $W > 0$

$$\lim_{t\to\infty} \pi_K(W,t) = \pi_K^* \quad (4.10)$$

Proof. According to (4.7) and (4.8), we get

$$\lim_{W\to 0} \frac{WV_{WW}(t_0,W) + (\gamma+\lambda)V_W(t_0,W)}{W^{-\gamma-\lambda}} = 0$$

$$\lim_{W\to\infty} WV_{WW}(t_0,W) + (\gamma+\lambda)V_W(t_0,W) = 0.$$

For any fixed $\epsilon > 0$, there is $\delta = \delta(\epsilon) > 0$, such that

$$|WV_{WW}(t_0,W) + (\gamma+\lambda)V_W(t_0,W)| \leq \epsilon W^{-\gamma-\lambda} + \delta, \forall W \in R_+. \quad (4.11)$$

Define

$$w(t,W) = \pm(WV_{WW}(t,W) + (\gamma+\lambda)V_W(t,W)) + \epsilon(W^{-\gamma-\lambda}e^{\xi(t-t_0)} + 1) + \delta e^{-r_B(t-t_0)},$$

for $\forall (t,W) \in [t_0, t_1] \times R_+$ with any $t_1 > t_0$. Then w satisfies the following equation and boundary conditions

$$\begin{cases} \frac{\partial w}{\partial t} + \frac{1}{2}\sigma_\omega^2 W^2 w_{WW} + (\phi + \sigma_\omega^2 W) w_W + \phi w = \phi\epsilon, & (t,W) \in (t_0,t_1) \times R_+, \\ w(t_0,W) > 0, \quad W \in R_+, \\ liminf_{W\to 0} w(t,W) > 0, liminf_{W\to\infty} w(t,W) > 0, & t \in [t_0,t_1]. \end{cases} \quad (4.12)$$

By the maximum principle, we conclude that $w(t,W) > 0$ for all $(t,y) \in [t_0,t_1] \times R_+$, which implies

$$|WV_{WW}(t,W) + (\gamma+\lambda)V_W(t,W)| \leq \epsilon(W^{-\gamma-\lambda}e^{\xi(t-t_0)} + 1) + \delta e^{-r(-\gamma-\lambda)(t-t_0)} \quad (4.13)$$

Let $t \to \infty$ and [illegible] 0, we have

$$\lim_{t\to\infty} |WV_{WW}(t,W) + (\gamma+\lambda)V_W(t,W)| = 0 \quad (4.14)$$

By (4.1), we can arrive equation (4.10). This ends the Proof of Theorem 4.3. □

Example 4.4. Define

$$u(c,W) = \frac{1}{4}(W-c)^{-4} + W^{-6}$$

A simple calculus gives that the utility functions above satisfies the conditions of Theorem 4.3 with $\gamma = 3$ and $\lambda = 2$, and (4.10) holds under this utility function.

On the basis of the optimal investment strategy in the equilibrium market obtained in the previous chapter, this chapter proves that under a kind of general utility function which is connected with the original power utility function, we can also get the investment strategy similar to the original portfolio optimization problem. In other words, under this kind of generalized utility function, the optimal strategy is to invest the fixed proportion of wealth in risk assets and risk-free assets respectively. On the premise of these assumptions, we use the optimal investment strategy to estimate the expected rate of return of bonds and equity in the market, describe the risk-free rate of return, and calculate the level of equity premium by changing the energy, so that we can connect the approximation of the optimal strategy with the approximation of equity premium in the following, which can explain the mystery of equity premium under the general utility function.

V. The explaination of the equity premium puzzle

We will explain the equity premium puzzle in this section. With the investor's optimal strategy, we can calculate the equity premium that the investor accepts. For simplicity, we set consumption tax,τ_c, to zero. With the optimal strategy that we got in the previous sections, the return rate on the market portfolio $Q = n_B W + n_K W$can be represented as

$$r_Q = r_B n_B + r_K(1-\tau)n_K = \alpha(1-\tau) - \frac{\alpha^2 W V_{WW}}{V_W}[\tau' \sigma_y^2 + \sigma_z^2]$$

Then we have

Propositin 5.1. The equilibrium asset-pricing relationships are

$$r_B - \frac{\eta}{WV_W} = \beta_B(r_Q - \frac{\eta}{WV_W}) \quad (5.1)$$

$$(1-\tau)r_K - \frac{\eta}{WV_W} = \beta - K(r_Q - \frac{\eta}{WV_W}) \quad (5.2)$$

where

$$\beta_B = \frac{cov(d\omega, du_B)}{var(d\omega)} = \frac{(1 - n_K(1-\tau'))\sigma_y^2 + \sigma_z^2}{n_B(\sigma_y^2 + \sigma_z^2)}$$

$$\beta_K = \frac{cov(d\omega, du_K)}{var(d\omega)} = \frac{(1-\tau')\sigma_y^2}{\sigma_y^2 + \sigma_z^2}$$

*Proof.*From Proposition 3.1.and equation (3.11),we have

$$\frac{\eta}{WV_W} = \alpha(1-\tau) + \frac{\alpha^2 W V_{WW}}{V_W}(1-\tau')\sigma_y^2$$

Since

$$r_Q = r_B n_B + r_K(1-\tau)n_K = \alpha(1-\tau) - \frac{\alpha^2 W V_{WW}}{V_W}[\tau' \sigma_y^2 + \sigma_z^2]$$

we get

$$r_Q - \frac{\eta}{WV_W} = -\frac{\alpha^2 W V_{WW}}{V_W}[\sigma_y^2 + \sigma_z^2]$$

and using Proposition 3.1. ,we can come to the conclusion. □

Again, $\frac{\eta}{WV_W}$ can be regarded as the risk-free return.The equation (5.1) and (5.2) shows that the returns on government bonds and capital are given by the familiar consumption-based capital asset pricing model with $r_Q$as the return on the market portfolio.

From the conclusions in Section 4, we know that as long as the investment horizon is sufficient long, the equations in Propositin 5.1. can converge to:

Corollary 5.2.

$$r_B - \frac{\eta}{\delta(1-\gamma-\lambda)W^{1-\gamma-\lambda}} = \beta_B(r_Q - \frac{\eta}{\delta(1-\gamma-\lambda)W^{1-\gamma-\lambda}}) \quad (5.3)$$

$$(1-\tau)r_K - \frac{\eta}{\delta(1-\gamma-\lambda)W^{1-\gamma-\lambda}} = \beta_K(r_Q - \frac{\eta}{\delta(1-\gamma-\lambda)W^{1-\gamma-\lambda}}) \quad (5.4)$$

where

$$\beta_B = \frac{cov(d\omega, du_B)}{var(d\omega)} = \frac{(1-n_K(1-\tau'))\sigma_y^2 + \sigma_z^2}{n_B(\sigma_y^2+\sigma_z^2)}$$

$$\beta_K = \frac{cov(d\omega, du_K)}{var(d\omega)} = \frac{(1-\tau')\sigma_y^2}{\sigma_y^2+\sigma_z^2}$$

We define $\overline{r}_B$ and $\overline{r}_K$ as the return rate on the government bond and the capital stock respectively in the absence of the spirit of capitalism, i.e. $\lambda = 0$. Like Gong and Zou [13],we can get

Propositin 5.3. If the investment horizon is sufficient long , and $\lambda > 0$, then we have

$$r_B - \frac{\eta}{\delta(1-\gamma-\lambda)W^{1-\gamma-\lambda}} > \overline{r}_B - \frac{\eta}{\delta(1-\gamma)W^{1-\gamma}}$$

$$r_K(1-\tau) - \frac{\eta}{\delta(1-\gamma-\lambda)W^{1-\gamma-\lambda}} > \overline{r}_K(1-\tau) - \frac{\eta}{\delta(1-\gamma)W^{1-\gamma}}$$

Propositin 5.3 indicates that the existence of the spirit of capitalism makes the gap between the risky assets and risk-free assets enlarged in the long term, and this explains the equity premium puzzle in Mehra and Prescott [1] partially.

Remark 5.4. Considering the difference caused by the spirit of capitalism in original papers($r_B - \frac{\eta}{\delta(1-\gamma-\lambda)W^{1-\gamma-\lambda}} - [\overline{r}_B - \frac{\eta}{\delta(1-\gamma)W^{1-\gamma}}]$),we can get the same difference when$u(c,W) = \frac{c^{1-\gamma}}{1-\gamma}W^{-\lambda}$. Since we study the problem under a wider range of utility functions, our result is obviously better than the original.

Example 5.5. Next, we will use our model to estimate the premium in Chinese market. From Proposition 3.1. and equation (3.11), we know that under the utility function $u(c,W) = \frac{c^{1-\gamma}}{1-\gamma}W^{-\lambda}$, the premiums of the capital stock and the government bond are calculated as follows:

$$(1-\tau)r_K - \frac{\eta}{\delta(1-\gamma-\lambda)W^{1-\gamma-\lambda}} = \alpha^2(\gamma+\lambda)(1-\tau')\sigma_y^2 \quad (5.5)$$

$$r_B - \frac{\eta}{\delta(1-\gamma-\lambda)W^{1-\gamma-\lambda}} = \frac{\alpha^2(\gamma+\lambda)}{n_B}[(1-n_K(1-\tau'))\sigma_y^2 + \sigma_z^2] \quad (5.6)$$

Now we choose A-share data in Chinese market directly from Shanghai Stock Exchange as the research object. Substituting Chinese market data into formula (5.5),(5.6) and choosing $\gamma = 3$,$\lambda = 0$, we then estimate the investor's acceptable premiums of bonds and A-share in Chinese market as table 1.

year	2011	2012	2013	2014	2015	2016	2017	2018
bonds	3.70%	5.57%	5.14%	4.02%	2.72%	2.88%	2.43%	4.39%
A-shares	4.22%	6.63%	5.85%	5.10%	3.53%	3.68%	3.01%	4.78%

table1 :Investors' acceptable equity premium in the past eight years when $\lambda = 0$

Similarly, if we select $\gamma = 3$,$\lambda = 1$, we have that the investor's acceptable premiums of bonds and A-share in Chinese market are shown in the following table 2.

year	2011	2012	2013	2014	2015	2016	2017	2018

bonds	3.80%	5.66%	5.53%	4.26%	2.89%	3.21%	2.79%	4.98%
A-shares	4.52%	6.83%	5.99%	5.31%	3.83%	3.91%	3.31%	5.57%

table2 :Investors' acceptable equity premium in the past eight years when $\lambda = 1$

This calculation result shows that the spirit of capitalism makes the gap between the risky assets and risk-free assets enlarged, which therefore confirms Propositin 5.3.

VI. Conclusion

In this paper, similar to the work by Gong and Zou [13], we study the equity premium puzzle based on the domestic output process. We further study the approximation of the optimal strategy. We proved that the optimal strategy under general utility functions satisfying certain conditions approximate the strategy under the original special utility function. Then we explained the equity premium puzzle.

VII. Acknowledgement

This work is supported by NNSF of China(No.11871275; No.11371294)

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